Teaching mathematics and social justice: multidimensionality and responsibility

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Introduction

This paper draws from a larger longitudinal study that follows approximately 1000 students over four years in three schools in California, USA. The study focuses on interactions among curriculum, teaching and learning, and tries to understand how particular teaching approaches influence learning. Our assessment, questionnaire and interview results show that at one school, which we call Railside, students achieved better results than students at the other two schools. This positive result is particularly interesting given the fact that Railside is an urban school, serving predominantly low SES students. The students at Railside also developed more productive dispositions towards mathematics and towards each other. They persisted more when faced with challenging problems and reported greater enjoyment of math class. Moreover, students developed the idea that mathematics is about communicating, explaining and justifying ideas, they viewed collaboration with others as central to mathematical work, and they held themselves accountable for helping each other to learn. The Railside teachers explicitly teach an approach to learning mathematics that includes responsibility to others. The innovation of their approach is the development of 'multidimensional' classrooms (Rosenholtz & Wilson, 1980) in which they focus on both meaningful mathematics and social justice.

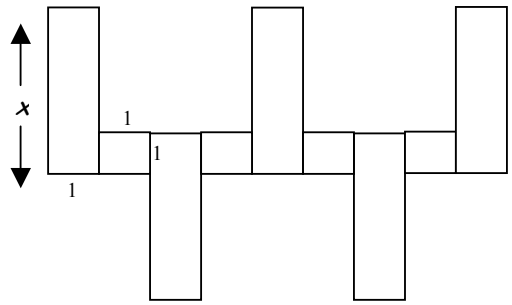
The above results are discussed in detail elsewhere (Boaler, 2003; Boaler, Brodie, Lerman & Zevenbergen, 2004). In this paper we undertake a detailed analysis of an episode of teaching and learning at Railside to try to understand the particular teaching practices that produce their success. In order to understand the teaching practices at Railside, our team of researchers spent many hours at the school, observing and videotaping lessons of 10 mathematics teachers, interviewing teachers and students, and participating in mathematics events such as their Algebra Week and mathematics field trips. We have selected this particular episode to write about because it clearly illustrates many of the principles and practices that we have seen during our time at the school: a commitment to equity and social justice, a commitment to enabling all students to progress to higher levels of mathematics at school and in college; a commitment to

finding ways in which students can grapple with and make sense of important mathematical ideas; and a commitment to collaboration and accountability among students[1].

The task

The teachers at Railside have developed an introductory algebra curriculum in order to meet the needs of their students, many of whom had not been successful with more traditional approaches in the past, and who enter the school with weak mathematical knowledge. The curriculum emphasizes connections between multiple representations of algebraic expressions. In the episode we discuss, the teacher and three students interacted around the following task:

Build the arrangement of LabGear™ blocks, and find the perimeter of the arrangement. Combine like terms in your answer.



The LabGear™ blocks are intended to enable students to build meaning for algebraic expressions. In this task, the blocks support students in recognizing and combining "like terms," and provide a geometric model for the subtraction of a number from a variable. The LabGear™ apparatus also functions as a support for collaboration, by providing students in a group a common focus and referent. Throughout this episode, the three students pointed to, manipulated, and wrote around the figure that they had built, which sat between them in the center of the table.

This episode began after the three students, Ana, Paulina, and David, had solved the problem and told the teacher that the answer was 10x + 10, which is correct. The teacher then asked Ana to show her where 10 is in the diagram, which became the focus of the subsequent interaction. In this interaction we see three key teacher practices: asking significant

[1] We note as well that the teachers are committed to working as a team to achieve these goals, so they promote collaboration among themselves as well. A discussion of this is beyond the scope of this paper.

mathematical questions; enabling collaboration; and keeping the demand high. These practices, together with others that we have identified elsewhere (Boaler, 2003) come together at Railside to create multidimensional classrooms (Rosenholtz & Wilson, 1980). These are classrooms which create a range of ways in which students can be successful while engaging in meaningful mathematics. In the rest of this paper, we discuss these practices, and how they contribute to multidimensional classrooms in which students are developing productive mathematical dispositions and a sense of responsibility for each other, while learning mathematics.

Asking significant questions

The question: *Where is 10?* required the students to link the algebraic and geometric representations of perimeter. We have heard many questions of this type asked by different teachers in this school, questions that focus students onto the connections between representations. Such questions also give students the opportunity to communicate their thinking and to learn, with teacher feedback, what constitutes a sufficient explanation. In this case, the question *Where is 10?* required the students to articulate the subtraction required to get the perimeter. The students would have to explain that they had to subtract 8 from 18, and that the subtracted 8 came from the eight pieces that look like they have a length of x, but actually have a length of (x-1). The question thus reinforced the opportunity in the task for the students to build meaning for subtraction in algebraic expressions by connecting the expression x-1 to a geometric representation. The same task, with a different follow-up question (or with none) would most probably have been less effective. Once the students had successfully answered this question, the teacher asked a second follow up question: *Where is 10x?*, again focusing the students on the links between the expression and the drawing.

Enabling collaboration and accountability

At the beginning of the episode Ana could not answer the teacher's question *Where is 10?* In this situation, many teachers would either help Ana to answer the question or ask one of the other students to explain. This teacher did neither; instead, she left the group to work together to help Ana understand the mathematics well enough so that she could explain it. This teaching practice is common at Railside. In all of Railside's math classes, which are untracked, students work in heterogeneous groups. In order to promote equity in these very diverse classrooms, the math teachers enact a version of Complex Instruction (Cohen & Lotan, 1997), through which they hold students accountable for each others' learning. The teachers establish a practice that no

one in a group is finished with a task until each member can explain and justify the answer. The fact that Ana could not explain meant that the group was not finished. Railside teachers see this practice of group accountability as helping to produce equity, in that all members are seen as crucial to the group, and all have to understand the mathematics well enough to explain it.

The teacher came back to this group a second time, and although Ana did better than the first time, her explanation was still incomplete. Ana asked the teacher: *Why are you asking me, why don't you ask them? He's the one who can explain*. The teacher stood her ground and answered: *So listen to him*. In this way she continued to position Ana as responsible for understanding David and Paulina's explanations, and David and Paulina as responsible for communicating their thinking clearly enough that Ana could learn from them. The third time the teacher returned, Ana explained clearly and competently, and the teacher reinforced the value of the students' collaboration by saying: *You need to make sure you keep everyone together. That everyone understands*.

Keeping the demand high

Each time the teacher came to work with this group, she could have simplified her question by giving hints where to look on the diagram or by asking questions that would have led them through their solution process. Research has shown that teachers tend to reduce task demands when students struggle (Stein et al, 2000). This teacher did not. In her first two interactions with the group, she asked 14 questions and none of them reduced demand. She repeated the question: *Where is 10?* a number of times. She also asked questions such as: *Wait, I'm confused. Did you show me the 10 or did you show me the 10x? When you go, "nine, ten"- what are you pointing at?* These questions are different from other questions that we have seen where teachers ask "why" and do not go further. Railside questions respond to the students' own thinking and focus upon areas of student confusion, forcing the students to be more precise in their meanings. Railside teachers keep both mathematical and collaborative demands high, which requires students to persist both in making personal sense of the mathematics and in explaining it to each other.

Multidimensionality

Asking significant questions, fostering collaboration, holding students accountable and keeping demand high support the creation of multidimensional classrooms. Multidimensional classrooms are those in which students can be successful in many different ways. When teachers

at Railside are asked about particular students' achievements, they talk not only about students' mathematical performance, but also about how students contribute to the understandings of others. In this episode, being successful meant not only arriving at the correct answer, but also explaining clearly to the teacher and other students, learning from other students, and asking questions that supported others in explaining more carefully. This group had the correct answer at the start of this episode, but Ana could not explain it, and explaining was maintained by the teacher and the students as an important part of Ana's, and the group's, success on the task. Until Ana could explain, no one in the group was fully successful. The above is in contrast to *uni*dimensional classrooms, where there is only one way to be successful, usually by executing procedures correctly and fast. This means that some students rise to the top and some sink to the bottom, and every student knows which kind of student they are.

It took some time for Ana to become able to explain, and each member of the group persisted in that part of their task, while also each learning from their conversation. As the students interacted to support Ana, it became clear to Paulina that she too wasn't quite sure where the 10 came from. As Paulina persisted in developing her own understanding, she supported David to articulate his thinking more carefully, and also enabled Ana to confront her own partial understanding and to ask questions which would get her the information she needed in order to think more deeply about the diagram. At one point Paulina asked *"minus eight, which are these?"* and David responded: *"No, no you just tell her minus 8."* Ana's response to this is: *"She's going to ask me where I got the eight from."* Ana appropriated the teacher's question to encourage David and Paulina in supporting her. In this brief interaction Ana, David, and Paulina were each successful contributors to the group's learning and subsequent success on the task, albeit in different ways. The teacher also contributed to this by asking a mathematically significant question requiring the students to make connections and explain their thinking; by refusing to simplify their task; and by encouraging collaboration.

Conclusion

The teachers at Railside have a commitment to teaching mathematics for equity and social justice. For them, as for many teachers, this means providing *all* students access to meaningful mathematics at school, the possibility for further study in mathematics and creating classrooms in which there are multiple pathways to success. It also means helping students develop both a productive disposition towards mathematics, which includes a willingness to persist when faced with challenging problems and despite initial failure, as well as a feeling of

responsibility for each others' learning. In the above analysis of one episode we have begun to show how their aims are translated through their practice into students' learning.

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