

SOME TENSIONS IN MATHEMATICS EDUCATION FOR DEMOCRACY¹

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Abstract

In this paper, I discuss some links between mathematics education and democracy, what these links could imply to what and how we teach, and the issues that arise from trying to further these links. I first suggest three links between mathematics education and democracy formulated on the basis of experiences in Denmark, in particular: learning to relate to authorities' use of mathematics, learning to act in a democracy, and developing a democratic classroom culture. The first two are discussed in relation to narratives from real life, with a focus on the tensions which they reveal. From the discussion following the first narrative, it is clear that what is a competency in one context may not be so in another. This is supported by the second narrative which also questions what is most relevant to students in South Africa and thereby gives rise to the formulation of a fourth connection between democracy and mathematics education, related to issues of access. The third narrative informs a discussion of what it means to be critical. It also continues to address the potential tension between wanting to promote students' critical skills and a democratic classroom culture versus wanting to support students in learning what others have developed and what is required in order to succeed in the schooling system. Finally, democracy is linked to the idea of 'mündigkeit', or 'personal authority'. This is not only an issue in relation to the students, but also in relation to teachers. On this basis, I briefly touch on teachers' possibilities for making choices concerning what and how to teach. This comprises a fifth connection between democracy and mathematics education.

MATHEMATICAL COMPETENCIES FOR DEMOCRATIC PARTICIPATION

Werner Blum and Mogens Niss (1989) and Niss (1987) list five groups of arguments for introducing modelling and applications into the mathematics curriculum. Though not mutually exclusive, they reflect very different goals for education, obviously involving socio-cultural/ideological values (Ernest, 1991; Niss, 1987: 6). The two goals most directly linked to democracy appear to be:

- Promote and qualify a critical orientation in students towards the use (and misuse) of mathematics in extra-mathematical contexts.

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- Prepare students to be able to practice applications and modelling – in other teaching subjects; as private individuals or as citizens, at present or in the future; or in their future professions.

The first goal is relevant because of the wide range of contexts in which mathematics is applied, where the purpose of the modelling may vary (descriptive, predictive or prescriptive, cf. (Davis & Hersh, 1986/88: 115-121)) or the nature of the foundation of the model may vary (from theoretically very strong to consisting of a collection of rather ad hoc assumptions, cf. (Emerik, Gottschau, Karpatschhof, Møller, & Nørgård, 1981; Jensen, 1980)).

The identity of methods and procedures masks the total diversity of situations and encourages the indiscriminate use of certain non validated mathematical methods in totally unacceptable contexts as well as in acceptable, productive contexts. (Booß-Bavnbek & Pate, 1989: 168)

For an in-depth discussion of types of models, the necessary types of critique and its relevance to mathematics discussion, see (Christiansen, 1996). Included in the second option may be the use of mathematics to create awareness of what for some would be considered problematic societal situations, as probably most strongly demonstrated in the activities for adult learners discussed by Marilyn Frankenstein (Frankenstein, 1981, 1983, 1990). A third obvious link between mathematics education and democracy is the development of a democratic and egalitarian culture in the classroom (cf. Ellsworth, 1989; Young, 1989). As Povey (2003) states it:

To harness mathematical learning for social justice involves rethinking and reframing mathematics classrooms so that both the relationship between participants and the relationship of the participants to mathematics (as well as the mathematics itself) is changed (p. 56).

For the purpose of this paper, the first two points, which so obviously involve mathematical competencies and using mathematics as a thinking tool, will be my focal point.

First Narrative: Mathematical Competencies in Critique of Models

A month before Christmas 1999, the summaries of the Danish news contained the following:

CITIZENS CHEATED OF 43 BILLIONS?

A 74 year old pensioner, Hans Peter Scharla Nielsen, today summoned the Ministry of Finance, demanding payment of an amount which the state wrongfully has withheld from him – and in his opinion from all other citizens in Denmark. He has meticulously studied the financial models which the Ministry of Finance uses in calculating the key numbers from which pensions, social security, and unemployment benefits are calculated – as well as the different taxation limits. His claim is that the Ministry of Finance since 1996 systematically has used misleading numbers for the average salaries in Denmark. Instead of looking at all salary earners and their salaries as a basis for calculating the average salary, the Ministry of Finance has – in collaboration with the Danish employers' association which provides the salary information – chosen to disregard IT companies. Also, they do not make corrections to make up for that several employees go from being appointed on a group contract

basis to being officials. The lower average implies that the pensions, social security, and unemployment benefits, which according to the law must follow the average, are too low. Hans Peter Scharla Nielsen thinks that it is an amount around 3 billion Dkr. At the same time, the limits for top and bottom taxation have been set too low – and that implies that the tax payers have paid app. 40 billions Dkr too much!

Hans Peter Scharla Nielsen is not some arbitrary pensioner – he has done it before! In 1996 he obtained judgement that the Ministry of Finance had calculated the pensions incorrectly and paid out 1.5 billion Dkr too much. That led to a reprimand to the Minister of Finance Mogens Lykketoft (Social Democratic party) and to the reduction of pensions for the following years.

There are many examples that mathematics plays a part in decision processes; cases where it requires a good deal of mathematical competencies to reflect critically on the situation (Blomhøj, 1999). In this case, however, one could claim that it is not what we usually understand as mathematical competencies which make it possible for HPS Nielsen to criticise the existing financial models. He 'simply' considered if all information had been utilised 'correctly'. However, that investigation required the use of numerical values and models, and it required a knowledge that information does not exist in and by itself but is constructed as part of the modelling process – choice of variables, formulation of connections and relations, the determination of constants, etc. It is the same kind of understanding of the choices underlying a modelling process involved in challenging the classification of research with military purposes as 'research' rather than as 'military spending' (Frankenstein, 1983).

Should we expand our understanding of 'mathematics' to include these core modelling competencies and the ability to relate in a critical fashion to models which involve mathematics? In the end, technology has to a large extent made it superfluous to learn the methods and techniques which for so long have dominated most mathematics instruction. But it still takes people to formulate and develop mathematical models and to interpret these as well as to apply and criticise the interpretations. As suggested by Ole Skovsmose (1990), the Ministry of Finance's application of mathematics may well influence who feels competent criticising the decisions from the Ministry – even if it is not the mathematics which makes a difference but the modelling process and its underlying assumptions. In other words, the use of mathematics may exclude someone from (feeling confident) taking part in the discussion. It may also change what we (believe) count(s) as arguments (Christiansen, 1996, 1998; Skovsmose, 1990). It remains an open question what the links between mathematical competencies, a general understanding of the limitations of the modelling process derived from specific experiences, and self-confidence are in determining a person's competencies and willingness to challenge political decisions as Hans Peter Scharla Nielsen did.

In Danish society, the use of mathematical models and scientific investigations play a role in legitimising certain political decisions – discussed by among others Morten Blomhøj (1999) and Peter Kemp (1980). The 'expert ideology' makes language games which focus on 'correct versus wrong' and 'efficient versus inefficient' more acceptable than language games which focus on 'ethical versus unethical' or 'esthetically pleasing versus esthetically offensive'. This also applies when mathematics is applied to realistic situations in the

classroom (Christiansen, 1996, 1997, 1998). But as Paola Valero has pointed out, this is far from the case in many other countries, such as Colombia:

[...] decisions are made based [...] also on personal loyalty [...], political convenience, power of conviction through the use of language or violent, physical imposition. In this political scenario and 'rationality', mathematics does not necessarily constitute a formatting power that greatly influences decision making. (Valero, 1999)

Is a focus on giving students competencies with which to critically consider mathematical models and their use really that relevant in the rest of the world? Is this focus relevant in Denmark – or is it more about becoming aware of and critical towards which discourse is the dominating one? Could and should mathematics education contribute to the development of this competency, simply because mathematics often is a tool in the ideology which relies on expert statements?

Double Purposes in a Task: Furthering Societal Awareness and Learning Mathematics as a Thinking Tool

Inspired by among others Marilyn Frankenstein, who again bases her work on Paolo Freire's ideas, I have tried to use examples in mathematics instruction on all levels. Examples which, in my opinion, would encourage students to use mathematics at a 'thinking tool' in critically considering a number of issues in society. Examples which at the same time could assist students in developing an understanding of the mathematical tools and how they may be used purposely. One such example was about distribution of land on 'races' in South Africa. I used it first in the teaching of student teachers in Copenhagen, with the following task formulation:

The table shows the share of land which 'blacks' and 'whites' in South Africa owned in 1981.

South Africa is 1.223.410 km²

	Population (<i>millions</i>)	Share of land (<i>percentage</i>)
'Blacks'	18	13
'Whites'	4	87

How would you illustrate this together with your students?

Which impression of the distribution of land do different methods of illustration give?

One can draw pie charts showing the distribution on races of land and population and experience that they look 'opposite'. This is where many of the Danish student teachers started. One can illustrate it by dividing the classroom space into two parts where one is 13% of the total space, and then distribute 22 students, each representing a million citizens. This would activate students and make it appear very 'real' to them. But that gives the misleading image that all the 'blacks' live on top of each other. Instead, one can calculate area per person:

There are app. 8700 m² per 'black' citizen in South Africa.

There are app. 260.000 m² per 'white' citizen in South Africa.

So a 'white' in average owns app. 30 times as much land as a 'black' does.

And one can go out to a field and measure out how many soccer fields is equivalent to the land owned by each 'black' in average.

But is all land in South Africa owned by someone?

For comparison (1988 data):

There are app. 4300 m² per citizen in the United Kingdom.

There are app. 39000 m² per citizen in the USA.

There are app. 394000 m² per citizen in Canada.

So in the United Kingdom, people live closer together than the 'black' South Africans do. And only in Canada is there more land per person than per South Africa 'white', in average. Still, these data say nothing about the quality of land owned by 'blacks' and 'whites' respectively. And we have not discussed differences within these two broad groups in the population. There is much more to discuss!

Which purposes can such an example serve in a mathematics class? It can make students

- Work with changing relative numbers to absolute numbers, and the other way around
- Illustrate numerical data, for instance illustrate a relative distribution in a pie chart
- Consider the message conveyed by different methods of illustration
- Discuss what quantitative information can and cannot tell us
- Obtain a basis for being critical towards others' use of mathematics and quantitative presentations
- Become aware of a political situation
- Challenge their view of all of Africa as poor
- Have their interest for learning mathematics stimulated through experiencing it as a 'thinking tool'
- Be actively involved in the teaching-learning situation

I thought that this example would have great potentials in a mathematics instruction (also directed towards furthering competencies of relevance in a democratic society. I did think that it would be a bit 'touchy' to use the example in South Africa. However, critical situations, critical questions and challenges, dilemmas and dialogue are core elements in an instruction with these critical intentions (cf. Vithal, 2003), as they are bound to raise issues which bring these competencies into play. Therefore, I used the example when visiting the former University of Durban-Westville (now merged with other institutions to form University of KwaZulu-Natal).

Second Narrative

In the class I visited, there were both 'african' (mostly Zulu) and 'indian' students. The 'indian' students started on the task, though they did not seem pleased. The 'african' students refused to take up the challenge – they could only be talked into starting on another task in the set. After the lesson, I had a conversation with students from both groups about their resistance towards the task.

The answer was pretty clear. Firstly, they did not like to be reminded of their situation. Rather than seeing the data as information which could be used in political argumentation, and the task as an exercise in working with data to support an argumentation, they felt reminded that they were considered to be worth less than 'whites'. Secondly, they did not find it 'good' to spend time on working with this kind of example in school. It would not, they said, give their future learners the mathematical competencies they need in order to do well, to secure themselves an education or a job.

When I thought that the task would be relevant to the students, it rested on the assumption that knowledge can be used in (political) argumentation, and on the assumption that competencies in presenting numerical data are useful in this connection. I have already raised the question whether these assumptions hold true, here or anywhere. But the narrative also raises questions about what is considered relevant by the students. In two ways.

After all, there is no reason to document the extreme inequalities to 'black' students – they know these far too well from their own experiences. Frankenstein points out that inequality becomes more apparent by being considered for entire groups of people (cf. Frankenstein, 1990), but here the extent of the inequality is known far beyond what the numbers document. The questions in South Africa at this time are different ones, and I did not come close to the core of the issue, at the same time as I did not give room for the students' primary experiences. (Not to forget, these students probably experienced me as 'white' and foreigner, which could have influenced their reading of the situation.)

So we must ask who has the right to define when something is a good example that can support the development of competencies in using mathematics. Often, it is assumed that students and teacher can agree what characterises a 'critical example' which it is worth addressing in the instruction. But it may not be so straight forward. For instance, Renuka Vithal (2003) found that the learners in a South African primary school were very critical towards decisions made at the school, while the teachers tried to pull them away from this type of critical examples and towards more general societal issues. Is content directed towards furthering democratic competencies worth anything if the power relations in the classroom remain the same?

Furthermore, in a society strongly dominated by inequality between different population groups, is it fair to want to further the students' critical competencies directed towards changing living conditions, with the risk that they are disadvantaged under existing conditions? A contradiction also pointed out by Paul Ernest as "personal empowerment versus examination success" (1991: 213), yet we must not ignore the ways in which examination success relates to subjective experiences of personal empowerment as well as to an 'objective' increase in power within existing structures. (Or are the students wrong – can the two be united? Did the task indeed not *also* involve the mathematical competencies?) This points towards a fourth connection between democracy and mathematics education, often recognised in the literature in equating mathematics with power, namely that mathematics education must make it possible for the less privileged to improve their lives. Because we cannot call it democracy if the differences in living conditions are outrageous!

To sum up, democracy refers to *formal* conditions concerning the interplay between the institutions of a democracy, *material* conditions concerning

distribution of goods and services, *ethical* conditions concerning equality, and finally conditions concerning the *possibility for participation* and re-action. (Skovsmose, 1994: 29)

The ways in which mathematics relates to these aspects may, as shown here, be in conflict with each other when they infuse educational goals.

Therefore: who has, when it comes down to it, the right to influence the purpose and content of education? Does our insistence on these 'critical examples' end up being 'imposition of emancipation'?

How would the historically advantaged feel if the educational system really came to function on the premises of the historically disadvantaged? If our cultural capital (Bourdieu, 1983/2004) was depreciated overnight? Would we not object to the purpose and content forced upon us – even if claimed to be emancipatory? If I had been to state my views before the Durban experience, I would have stressed that transformation is not mainly about access; it is about changing values. But whose values are to be furthered? And who am I, who have access (both to and gained via mathematics), to say that it is mainly about values?

Empowerment through Learning 'Pure' Mathematics?

Third Narrative

We are in a fourth grade in a school near the centre of Dallas, USA. It is an area where the mortality for young men is very high; where it is dangerous to be after dark because of, among other, gang fights; where drugs are sold on street corners; and where most children come to school without having eaten breakfast. All the learners in this class are so-called African-Americans.

The learners answer my questions and comment on each others' contributions orally or with hand signals. Both hands straight up in the air means 'I agree', moving the arms horizontally over each other means 'I disagree' and another signal means 'I am not sure'.

They work with addition of negative numbers. In the first lessons I had with them, they came to a standstill in their attempt to find a solution to

$$5 + \underline{\hspace{2cm}} = 0$$

and that gave me a basis for introducing the additive inverse. Thus the learners were introduced to -5 as the number you add to 5 to get 0. Addition of the negative numbers can then be introduced through a series of tasks and questions. Right now, I have given the learners the task to find the solution to

$$7 + -4 = \underline{\hspace{2cm}}$$

Most of them think either that the solution is 3 or that it is impossible – we can only add -4 and 4, nothing else is allowed. The leading question is whether they can see 4 somewhere. After some probing, they rewrite the expression using $7 = 3+4$, making the original equation into

$$3 + 4 + -4 = \underline{\hspace{2cm}}$$

It is easy for the learners to prove that the answer to this must be 3. And as the two statements are equivalent (we show that by drawing arrows from each symbol in one line to the corresponding symbol(s) in the other line) 3 must also be the solution to the original equation.

The lesson is almost over, so I ask the learners to join in praise of each other. Then a girl raises her hand. She tells us that she looked in her brother's maths book. Her brother is one of the few from this environment who has managed to get into college. "You know what? There is a whole chapter on negative numbers, but my brother cannot figure out how to do it."

I praise the students – imagine, they are doing 'college mathematics'!

The philosophy behind this American project, called Project SEED, is to give the learners self confidence through mathematics. Through special tasks and leading questions, they are to help each other in reasoning out mathematical rules and connections. That means that they not only master calculation rules which to most US-Americans seem like an almost endless row of meaningless algorithms. They also develop a first understanding of the structure of mathematics and of the mathematical processes, in particular conjecturing, proving and generalising. Thus, the students obtain competencies which are highly valued in their further education, making them more on par with students from more affluent backgrounds. Finally, the high value associated with mathematics mastery in the USA was thought to contribute positively to learners' self image.

Would this be more agreeable to the South African student teachers from Durban?

At no time did Project SEED question a system where learners are to master this type of mathematics, and where applications mainly are what William (1997) has called 'MacGuffins' - "... a plot device primarily intended to motivate the action in a film, and to which relatively little attention is paid". The power relation between learners and teachers were not altered either – except that we unlike the regular teachers were not allowed to hit the learners.

CRITICAL COMPETENCIES AND MATHEMATICAL CREATIVITY - LINKED?

In contrast to Project SEED, we have narratives from classrooms where the learners actively work with developing mathematics from a practical problem, related to their everyday lives or not, and where they work more on their own, with less probing and fewer leading questions. Such narratives we have from all over the world and from various types of settings. For instance, (Beck, Hansen, Jørgensen, Petersen, & Bollerslev, 1999) and (Slammert, 1993) tell how students (in Denmark and South Africa, respectively) have worked with the task of making three red and three green frogs change places in accordance with certain given rules. In both cases, the learners invented a notation which could help them remember what they did and make it object for systematic treatment afterwards.

These classrooms appear to be more democratic in the way learners and teachers interact (though the task was still chosen by the teachers), because there is more equal participation and the possibility of negotiation meaning (or even task). But what does it mean to the development of competencies? Does it give students experiences with developing mathematics, which both assist them in doing well in the existing educational system, and make them capable of relating critically to the use of mathematics and use

mathematics actively in their democratic participation? Or does it make mathematics harder for the majority and hinders that they experience the advantages of being familiar with existing algorithms? When is one type of learning preferable to the other?

Let us, however, for a minute assume that most students have intellectual potentials which are rarely realised in schools – for a wide number of reasons not discussed here. How does working with open-ended tasks (or even with project work which includes some elements of problem posing) promote democratic competencies?

If by critical thinking one understand the ability to consider what could be different – go from the actual to the potential – then it is a central ingredient in mathematical creativity. Lebesgue was capable of developing a new integral because he could go from looking at the function 'seen from the x-axis' to 'seen from the y-axis' (Lebesgue, 1966). Other examples are discussed in (Kitcher, 1984). See also (Mason & Watson, 1999).

In order to develop mathematics more or less on their own, without too many leading questions, the students have to make choices – in contrast to repeating established habits. To make choices necessitates taking a stance, and it necessitates awareness on what could be different, what has been taken for granted so far – a critical approach. Are these competencies – critical awareness, ability to make a stance, ability to make choices, independence – which can be transferred to other situations, or do they remain tied to mathematics if that is the context in which they were developed? And opposite: are they competencies which can be developed in other contexts or through the learning of particular tools/skills, and then applied in learning mathematics *as well as* in participating in society?

Are they competencies which we are to further at any time, or do they only have limited legitimacy? Are these competencies which all students can develop? How do we avoid that focus on these kinds of more demanding competencies in instruction does not in itself end up being discriminating?

If indeed such competencies are to be furthered, how does one go about it? Lionel Slammert writes about 'mathematical awareness': "the inner self-sensing ability of the learner to pay attention internally to mathematical information, whether it be in the form of a shape, a principle, a concept, a method, and/or a system" (Slammert, 1993: 118). And through examples, he suggests that instruction can further this mathematical awareness by working with open problems which contain potentials for mathematical creativity, and by the encouragement to direct attention to the feelings and sensations experienced when working mathematically. I would add to this: by challenging the students' approaches or point their awareness to alternatives.

Teachers as Instruments and Agents with Personal Authority

If instruction is to do this, among others, then that means that *teachers* must make it happen. What then is required of teachers if they are to choose suitable problems with the desired potentials, if they are to know when and how to challenge and direct, if they are to know when to direct students towards or tell them about standard algorithms, and so forth?

For teachers to be able to this, to make the classroom dominated by mathematical creativity, they themselves must be creative, both mathematically and pedagogically. They must be capable of thinking in alternatives, both mathematically and pedagogically.

In order to do this, teachers must be aware of mathematical potentials in tasks and student activities. This requires extensive mathematical knowledge. If teachers are to do this, they must understand why the algorithms work; they must be able to solve the same problem in

different ways; and they must have a good understanding of how concepts are connected, of mathematical structures (cf. Ma, 1999).

Similarly, teachers must be aware of pedagogical potentials in relation to both the individual student and the class as a whole. This requires substantial amounts of pedagogical knowledge as well as pedagogical content knowledge (i.e., knowledge about how students learn mathematics, about concept development, etc.).

Paradoxically, in order to free attention for all of this – mathematical and pedagogical awareness and creativity – teachers must also be able to handle elements of their teaching as routine – ‘through algorithm’. The same must be true if they are to be aware of pedagogical potentials or potentials in relation to the furthering of democratic competencies.

Fourth Narrative

The new mathematics curriculum in South Africa is formulated as a list of desired outcomes, both general and specific to the different learning areas. In addition, there are assessment criteria for the various outcomes, varying with level of schooling, some guidelines for the organisation of teaching, pedagogics, and much more. Schooling is supposedly free, but the School Governing Board can decide to require payment of school fees, an upper limit to which is determined by the lowest income of school goers' families. The fees are utilised in hiring additional teachers, paying for materials, etc.

I observed a teacher in a school in the Western Cape and conversed with her about her teaching. It was clear that she tried to teach in accordance with the new guidelines – according to a teacher educator, she mainly used the examples used in in-service training courses (person communication with Lynn Rossouw, University of the Western Cape). But the way in which she understood the guidelines, they could not be realised in her everyday practice. And perhaps she was most concerned about meeting what she thought my expectations were. Note I=Iben:

T: Did you ... did I ... meet up with ooh sorry ... with with what you are expecting?

A little later I asked about OBE. The teacher commented (referring to a R1 allowance per child she has for this class for a given period of time):

T: I find it is the OBE's very expensive [...] After that we have to buy our own paper

I: So you bought this paper and you bought these smarties [box of small chocolates]

T: Yes so I bought the smarties from their one rand and it's it's about seventy five cents a box

I: Okay

T: You understand so you cannot follow this method all the time

The vital part of democracy is not the possibility of voting as much as what comes before the casting of the vote. There must be real choices connected to making decisions, there must be a possibility to debate options in depth, and there must be awareness on the importance of these elements and thereby a willingness to open up to new perspectives and

possibilities; a willingness to change one's mind. Democracy requires 'mündigkeit' – personal authority: to have the right to have influence, to be able to speak for one's case, to be responsible for one's actions and agreements. It gives rights but also responsibilities. It puts one under the obligation to engage and to stay informed. It puts one under the obligation to know the actual situation but be open to consider the potentials contained in actuality.

This also holds for teachers in relation to their teaching and the given guidelines for what and how to teach – whereby the connection between democracy and mathematics education is supplied with another level. For teachers to act with personal authority implies being open to considering the potentials in the actual instruction as well as in new curricula, guideline documents, etc. It puts them under the obligation to be well informed and critical both mathematically and concerning pedagogical content knowledge. An obligation to object when a potentially empowering curriculum is countered by limited assessment criteria representing traditional values and hindering mathematical creativity. An obligation to object when teachers' space for pedagogical creativity is limited by recipe-like descriptions of how to teach.

In extension of these obligations, it must also be a democratic right for teachers to have a say in how curricula, guidelines and recommended teaching materials are put together; a right to have the many years of experience from the teaching profession being put to use. A right to be taken serious if they choose to criticise curricula and required teaching methods for being too idealistic and too demanding to realise in practice. Do we secure these rights?

Summary

The first part of the discussion in this paper was first intended as a message to the Danish mathematics education community: that we had been too ethno-centric in our view on the connections between democracy and mathematics education. Looking outside the country makes us realise the assumptions on which we base our discussions: that perhaps the reference to expert statements matter more some places than others, and forgetting that the inequalities in access to resources and education is rather different in other parts of the world.

Naturally, rethinking these points in the light of my South African experiences changed the focus. I found it necessary to develop the points about equality being important to democracy and about teachers' rights and obligations – both elements which to a large extent are assumed to be in place in Denmark, though it has been questioned by critical voices.

What now stands out is a very inclusive perspective on democracy and mathematics education. In my discussion, I have first considered democratic issues in relation to mathematics education as a whole – what we must teach in order to further democracy. One aspect hereof is to develop relevant competencies, so students can empower themselves to deal with authorities in society. Out of this comes the idea of using critical examples, in particular by including modelling and critical reflection on modelling in the mathematics classroom. This is, however, challenged by the other aspect, namely on furthering equality in access and living conditions. The question remains: how are we to balance these two considerations?

Not pretending to answer this, an additional twist was added by my discussion of mathematics itself, where I have suggested that critical thinking vital to acting in a democracy also play a central part in mathematics – though not in any respect implying that one will automatically lead to the other.

The question of 'how to teach' is a general educational issue. I only briefly mentioned the issue of a democratic classroom culture with participation and negotiation, where students empower themselves to deal with authorities in the classroom (fellow students as well as teachers). This has, I believe, been given sufficient attention elsewhere. Instead, I discussed the obligations this puts on the teacher to be pedagogically (as well as mathematically) creative. This revealed that democracy in relation to education is not only students being empowered, but also about teachers' empowerment.

I stressed the dual nature of personal authority, the rights it implies as well as the obligations it puts on you. If students are to exercise their rights in a democracy, they have an obligation to put an effort into it, including developing the necessary competencies through schooling (which needless to say implies a right to have access to appropriate schooling!). If teachers are to exercise their rights in a democracy, they have an obligation to put an effort into it, including developing the necessary competencies to engage in dialogue with authorities (which needless to say implies a right to be heard!). As stressed by Renuka Vithal (2003), authority and democracy are not exclusive, they are complementary – in opposition, yet necessary for each other's existence.

Post-script: The Researcher's Personal Journey Reflected in the Paper

The paper is also a selective biography with narratives from the three countries in which I have lived and worked. My experiences from living in Denmark and engaging with the use of mathematical modelling in our society were reflected in the first narrative. Teaching student teachers in Denmark and South Africa turned out to offer rather different perspectives, and that was the basis for my second narrative. The third narrative served more as a contrast, though it also revealed how much students can learn with strong guidance. It is formulated on the basis of my experiences from more than a decade ago, teaching in Dallas. When I came to South Africa for a longer period for the first time, I got engaged with research on actual practices in schools in the Western Cape. This is the origin of the fourth narrative.

The series of narratives reflects how a researcher's personal experiences and culture, as well as teaching and research activities, are linked, how they limit each other as well as contribute to the development of each other. It reflects how bringing one's experiences together through critical reflection will continue to add new understanding to past events. It weaves a network of meaning through my activities which can only be seen in retrospect. Thereby, it becomes a recreation of my lived narrative of my past and thus of myself, and in that sense it shapes my present and future as scholar, supervisor, lecturer, colleague, citizen and mother.

To me, writing a paper based on such glimpses from my life reflects a deep recognition that research is not an activity which is separated from one's values and from who the researcher is as a person, nor is it an activity which happens in isolation. Writing this paper based on personal experiences is trying to be as good as my word, as I have often argued that reflections on one's own experiences is indeed valuable contributions to the community. I hope the reader will agree with me on this.

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