

CHAPTER 4

THINKING ABOUT MATHEMATICAL LEARNING WITH CABO VERDE *ARDINAS*

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The research reported in this chapter takes part of the results of the project *Cultura, Matemática e Cognição — Pensar a Aprendizagem em Portugal e Cabo Verde*ⁱ, especially in what concerns understanding the idea of “learning as an integral part of generative social practice in the lived-in world” (Lave & Wenger, 1991, p.35) together with the perspective that addresses “learning as increasing participation in communities of practice” (p.49). The analysis of these ideas led us to try to understand the meaning of participation in a social practice (and therefore in a community of practice). Our goal was to look into the ways (mathematics) learning relates to forms of participation in social practice in an environment where mathematics is present but that escapes the characteristics of the school environment. Because we believe that culture is an unavoidable fact that shapes our way of seeing and analysing things, we decided to look at a culturally distinct practice and that constituted a really strange domain for us: the practice of the *ardinas*ⁱⁱ at Cabo Verde islands in Africa.

In order to address the research problem we looked for analytical tools that we believe are coherent with the theoretical perspective drawing from Lave and Wenger (1991) and our need to understand the idea of social practice: (i) the explicit presence of rules in the *ardinas* discourse led us to Wittgenstein (1992/1953) and Goffman (1991/1974); (ii) the need to clarify the relationship between the ways that *ardinas* use mathematical objects within the practice of selling newspapers and the school mathematics they are supposed to know led us to address the connections between competence and “taxonomic features of knowledge” (Julien, 1997) and to the idea of “thinking as internal conversation” (Restivo, 1998).

In this chapter we first set up the scene presenting a full description of the practice observed in Cabo Verde; the subsequent analysis takes advantage of short episodes depicted from the observations made; finally we conclude with a summary of our main findings.

1. A glance on the *ardinas* practice

The practice of the *ardinas* has naturally a history that relates to the evolution of the country and to the society where it develops. The data collection that supports this research was carried out in two time periods: the first phase was from March to June 1998 and the second phase during the month of March 1999. This way it was possible to enlighten the historical transformation the practice, mainly through focusing our attention on the transitions experienced by the participants. Focusing our analysis on those experiences helped us to understand better the forms of participation of the *ardinas* in that practice, its role in the learning that came out of that participation as well as in their use of mathematics. This research process brought also to the front some other issues that pushed us to reflect upon the research process itself within the field of mathematics education research.

Therefore, it seems important to give a picture of the life that we could share with the *ardinas* during the time of data collection, trying to make explicit the relationships among people, between people and activity and the lived-in world. We begin this part with a brief and global presentation of the *ardinas* involved in the study. In a second step we will present a description of the practice observed taking into account the time sequence of its development.

1.1. Who are the *ardinas*?

The *ardinas* are young boys aged between 12 and 17 years that sell newspapers in the streets of Praia (the capital of the Republic of Cabo Verde). In 1998 there was just one national newspaper (called *O Tempo*) but from January 1999 there appeared a new one (called *O Espaço*). Both of these newspapers come out once a week and are written in Portugueseⁱⁱⁱ. The group of *ardinas* who used to sell these two newspapers was variable (19 in 1998 and 32 in 1999). Only 9 *ardinas* from the 1998 group were carrying on this practice in 1999, and there was no formal link to the institutions that owned the newspapers.

One of the newspapers (*O Tempo*) was trying to implement a selling system based on the shops such as coffee shops or stationary shops but with very low success. In fact the population did not adapt to this way of buying newspapers, so selling newspapers in the city of Praia was totally dependent upon the availability and interest of the young boys to volunteer for selling. Some of these boys also were in charge, both in the past and in the present, of selling Portuguese newspapers, and especially a particular one on sports very popular in Cabo Verde. This one was on sale on the street but on a different day. There were also two other *ardinas* who did not sell national newspapers because they were dedicated to selling two Portuguese weekly newspapers of general interest whose contents are more in the field of politics. These two young boys had already a small number (but a rather constant number) of clients and the places for selling were clearly different from the others. In most cases these *ardinas* were taking the newspapers to the offices and hotels in the city. The relationships between these two *ardinas* and the others were not very strong and frequently they did not even say hello when they met in the street.

There was no external sign (such as a special t-shirt, a bag or a cap) that could help one to identify the *ardina* except the fact that he was carrying a number of newspapers under his arm. However they were careful in the way they dressed on the

days of selling. They managed to be clean and it seems that they tried to maintain a certain combination of clothes especially for that activity. Some of them had special care with clothing in order to have access to certain places of selling (for example, in official departments).

Some of the boys started selling newspapers just prior to the data collection for this project (March 1998) but others had already been selling newspapers in the street for about six years. Most of them were *ardinas* because they wanted to get some money to help the family ("*to help my mother*" as they usually said).

Because data collection was carried on in two phases we could identify differences in these two periods. In 1998, the group of *ardinas* was living in two places, 9 *ardinas* were living at the Eugenio Lima area in Praia, which was one of the most problematic places in the city mostly with inhabitants coming from the rural zones looking for a job in Praia. There was another group of 10 *ardinas* coming from S. Martinho, a small village close to Praia. In 1999 the group was enlarged with boys coming from Praia, and those from S. Martinho started leaving this activity^{iv}. The growing of the number of *ardinas* in the group was slow; for example, during the month of March 1999 we observed the integration of only one new *ardina*.

1.2. *Ardinas'* practice

In the two periods of data collection (1998 and 1999) some common aspects were observed; but we could also identify different aspects. The work of the *ardinas* was divided into three different phases: (i) receiving the newspapers, (ii) selling, and (iii) paying back the money to the newspaper agency. The organisation of these three phases was necessarily connected to the instructions of the directors of the newspapers but the *ardinas* positioned themselves in that organisation in their own way according to several facts (that then become more visible). This means that in this practice we could identify a pre-existent structure or arena, in Lave's (1988) terms, which was redefined in the local context of action and that develops with the participation of the *ardinas* (and that certainly would be different for other *ardinas*).

1.2.1. *In 1998*

Every Friday morning, in the main building of *O Tempo* agency, the newspapers were delivered to Disidori, the man who was responsible for the whole process of selling^v, returning the non sold newspapers and payment. In order to have the newspapers sold Disidori distributed them among the *ardinas*; this operation took place at the door of the agency. The number of newspapers distributed to each *ardina* was negotiated and in most cases the number of newspapers varied between 50 and 150 for each. During the **distribution** Disidori wrote down in a list the names of the *ardinas* and the number of newspapers distributed to each one. This list was the reference document for the final phase when the *ardinas* were paying back to Disidori after selling.

The participation of the *ardinas* in the activity of selling was based on their will to do that. The link of the *ardinas* to the newspaper agency was very informal assuming a very personal character in relation to Disidori (more than to the agency); there was no

penalty and no need for justification if the *ardina* decided not to show up for selling. If he decided afterwards to come back for selling he knew that he could do it (notwithstanding that he could not have immediately available the number of newspapers he wanted to have). On his side, Disidori had a link to the administration of the newspaper, which was made visible to all when he signed a document against the delivery of the newspapers (which made him responsible for the payment to the administration). Besides that, Disidori received a fixed amount of money (that he recognised as the payment for assuming the responsibility) plus a part of the money of selling each newspaper. We can say that there was a mutual dependence among the *ardinas* (in order to get money they had to sell the newspapers), Disidori (to get more money he had to be sure that the *ardinas* really sold) and the administration of the newspaper (to sell the newspapers they needed the help of Disidori and the *ardinas*).

The group of *ardinas* changed over time. After a varied period of time some of the *ardinas* abandoned the activity of selling newspapers. Usually these boys got involved in other activities (for example, serving the army, getting a job or emigrating to another country). New boys then came to substitute for those who dropped out and generally these newcomers were accompanying a friend or someone from his family who was already an *ardina*. In the case of these new *ardinas* Disidori had the last word for their acceptance in the group. Disidori tried to know the *ardina* and talk to his family. He also used to chose one of the old-timers to take responsibility on the newcomer – “to teach him and to protect him” as they said. In the distribution of the newspapers among the *ardinas* what happened was that the old-timer received all the newspapers (for him and for the newcomer) and it was his job to give a small number of newspapers to the newcomer (first 5, then 10, 15, etc.) as he was selling.

Immediately after receiving the newspapers the *ardinas* run very quickly to the usual places for **selling** in the city; their goal is to try to sell all the newspapers during the day. Some of the *ardinas* tried to maintain their own place of selling. However, those places varied during the day according to the rhythm of selling and the rhythm of the city (namely, at the street in the rush time, at the working places on the working schedule, close to the restaurants at lunchtime). The price of the newspaper for the customer was 100 escudos; by the end of selling, the *ardinas* should pay to Disidori 87.5 escudos per newspaper sold and give back the non-sold newspapers. These amounts were defined by the newspaper administration. During the day most of the *ardinas* spent some time at the Square of the city or in nearby streets given that these were the places where selling was more common. In fact this is the area where most of offices, banks and public services, coffee shops and markets are located. On the other side this is the zone of the city where some local people develop their activity selling sweets and pottering on the street. Because those people stayed on the Square during the day, the *ardinas* got a close relationship with them which is in fact useful to both parts: the *ardinas* ask those people to keep a number a newspapers for them avoiding to have to carry a big number of newspapers, they exchange small coins in order to facilitate the change to customers and together contribute to attract potential customers.

Besides the strategic role of that interaction in the integration of the newspaper selling into the socio-economic life of the city, the Square was the place where Disidori stayed for long periods during the day of selling. He also walked around to the different places where the *ardinas* were selling in order to check how the process was

going. Some time after the distribution of the newspapers by the *ardinas* Disidori went to the Square carrying with him a set of newspapers for the possibility of those *ardinas* who were in the school (and because of that could not come to the distribution of the newspapers at the agency) or that he could distribute to those who sell very quickly and ask for more newspapers. The Square was the main point of convergence of the boys at several moments during the day: (i) at lunch time, those who did not approach the restaurants to sell, stay and rest for a while, (ii) when they finished selling and come to pay back to Disidori.

Those *ardinas* who were in a beginning phase of learning the practice of selling usually kept close to an old-timer who was responsible for them. On one hand, this was because the newcomers received a small number of newspapers (which they had to pay back to the old-timer before receiving more newspapers for selling). On the other hand, it was within the observation and interaction with the old-timer that the newcomer started to understand important aspects of the activity (how to handle money and newspapers, which were the good places for selling, how to address the customers). Besides that, the newcomers learned also to identify potential situations of risk. Because the number of newspapers that the newcomer received was increasing, he could start going farther and farther away from the old-timer and this also contributed to his self-confidence and gain of courage to go through different streets, getting independent of the old-timer.

As the *ardinas* finished selling the newspapers (or got tired of selling) they started showing up in the Square where Disidori was awaiting them for the **payment**. The *ardinas* approached Disidori, they said how many newspapers were left or how many they had sold, Disidori made the calculation (87.5 times the number of newspapers sold) with his hand calculator, he showed the result on the screen to the *ardina* who then gave him the money. Sometimes those *ardinas* with more independence made their own calculation (with their calculator or Disidori's calculator); Disidori trusted their calculation but generally he confirmed by himself the number of newspapers left and returned by the *ardina*. This moment was lived in groups by the *ardinas*; the newcomers also participated in the process (even if they received newspapers from an old-timer and pay him back directly). Several operations were in progress: some of the *ardinas* were counting the number of newspapers left for returning, others were counting and organising the money according to the value of bills and coins, they delivered newspapers to Disidori, observed the calculation, give the money to Disidori. The environment could seem confusing at a first glance; there was a lot of money on sight changing from hand to hand. However, observing in detail one could understand that everything was running in a certain order and this allowed that each *ardina* could see what was going on with the calculations (their own or those of a colleague). This was one of the opportunities for talking about the happenings of the day: for example, the customers who did not pay for the newspaper, the customers who returned the newspapers after reading it (making it possible that the newspaper was sold again), the *ardinas* who had lost money or made mistakes with the change or even the case of some *ardina* who did not show up to pay for the newspapers sold. So these were real moments of learning for all of them, both because of the stories heard and the behaviours observed (in their peers and in Disidori) face to face situations that were explained and discussed, and also because of the different activities that were taking place at the same time and the way the *ardinas* interpreted and solved their own problems. It was curious that after paying Disidori and keeping their own money,

it was common that the *ardinas* did not go away immediately; they stayed observing what was going on or helping the others after seeing their case resolved. Another aspect that called our attention was the total absence within the ordinary *ardinas'* discourse of any attempt to make explicit (verbal explanation or deliberately showing) their calculation strategies, checking processes or anything we could classify as some sort of mathematical conversation. We will look more closely to this issue in the section 2.1.

On the Square Disidori did not limit his activity to controlling the *ardinas* or receiving the money for the newspapers sold. He tried to give them suggestions of good places for selling at a certain time of the day, he made observations about the behaviour of potential customers specially those travelling in cars and stopping at the lights. Disidori was helping the *ardinas* to read the signs of the city and of its inhabitants in order to allow the *ardinas* to sell more quickly and more efficiently. But Disidori also talked a lot with the *ardinas* on subjects that had nothing to do with the process of selling. In fact he behaved as an adult who was feeling a responsibility for the education of those young boys taking care that they did not get involved in illegal street games, or making comments about the way they spent the money they earned as *ardinas*. This is in fact the natural and traditional way that in Cabo Verde adults feel and act in relation to children.

1.2.2. In 1999

During the period of the second data collection (March 1999) there were two different newspapers on sale in the city, both coming out once a week: *O Tempo* and *O Espaço*. Now, the majority of the 32 *ardinas* were involved in the selling of both the newspapers. However a small number of *ardinas* were linked only to one of the newspapers; this was due to (i) the fact that the new newspaper had a different day and place for **distribution**, (ii) the distribution was made by different people in the two newspapers, and (iii) there was an explicit instruction on the part of the administration of the newspaper *O Tempo* calling the attention of the *ardinas* to the fact that they should not sell both newspapers together. This instruction was given to the *ardinas* in a meeting that the administration of the newspaper promoted with them. This meeting was on 24 December 1998 and the *ardinas* were invited to a Christmas party. This was the moment when Disidori moved to a new position at the new newspaper *O Espaço*. This was also the opportunity for the local television making a piece about the *ardinas'* life. But the party was also the moment for the administration to announce the new rules: now for each newspaper sold the *ardinas* would receive 20 escudos (instead of the previous 12.5 escudos) but they had to go to the newspaper agency to receive the newspapers and go back there after selling in order to pay for the newspapers sold.

During this second period of data collection it was possible to identify different forms of participation. For example, in the case of Bétu, he had already been considered a good *ardina* in 1998.^{vi} Now, besides selling he was also being paid by the agency (only in *O Tempo*) for his work of putting together the pages of the newspaper after the printing process. And he received a large number of newspapers for selling (150 to 200) that he distributed to a small number of *ardinas* that he advised in the selling process (his brother, his cousin and a neighbour). In this new form of participation,

the *ardinas* paid Bétu and he was responsible for the payment of the newspapers to the agency. But the other *ardinas* still saw Bétu as before: although he had a particular relationship to the newspaper agency, they didn't recognise any authority over them.

Another case was Manu, one of the oldest *ardinas*, recognised by all of them as a good *ardina* and someone whom Disidori trusted. In 1999 Manu was also selling newspapers in the street but he helped (only in *O Espaço*) the work of preparing the newspapers for selling after the printing of the pages and was in charge of part of the interface between the agency and the *ardinas*: he distributed the newspapers to them, received the payment, and remained in the Square of the city during the selling (such as Disidori did in 1998). His main responsibility was in relation to the *ardinas*. This means that immediately after the time to start distributing the newspapers he left the work in the agency and assumed his role in connection with the *ardinas*. His first action was to write down a table with four columns: Taken, Sold, Left and Paid^{vii} and each row was allocated to each *ardina*. This was his tool for control of the whole process and Manu made this record visible to all those who wanted to see it during the day. He updated the records in the case that some of the *ardinas* took more newspapers to sell, writing down again in a different row the name and the number of newspapers taken. This was also his record for the final account at the end of the day. Then Manu organised the money and the newspapers not sold in order to make the final account with Disidori. Disidori usually went to Manu's home at the end of the day to collect the money and the newspapers left and to organize the activity for the next day.

The *ardinas* recognised authority in Manu but they keep considering him essentially as an *ardina* and they had the perception that he had not substituted Disidori (“*he is just helping him*” as Kaka said) and that he was not a member of the staff of the newspaper agency. For them, Manu had a special status in respect to Disidori and so indirectly to the institution.

The organisation of the distribution of the newspapers was different in 1999. The distribution of *O Tempo* took place on Friday at the agency of the newspaper away from the centre of the city. There was the intention of having *O Espaço* on sale on Thursday (almost every time this was not achieved) and the newspapers were delivered to the *ardinas* in a building (close to the Square) where the newspaper was printed and organised for the distribution. *O Tempo* was delivered to the *ardinas* by the treasurer of the agency. Before having the newspapers delivered to the *ardinas* the priority was to the people who took the newspapers to other islands and to the shops where now they were also sold. Only after that the *ardinas* received the newspapers and they had two options: Taking or Buying, according to their own words^{viii}. The agency of *O Tempo* defined a priority for the *ardinas* who buy the newspapers against those who just take them for selling. The decision on who receives and how many newspapers are delivered was a responsibility of the treasurer (sometimes with the interference of the director) but no explanation was given about the reasons behind those decisions. We could notice that the *ardinas* from S. Martinho had to buy the newspapers. In general when one of the *ardinas* did not pay back the agency for the newspapers taken, he had to come to the regime of buying instead of taking. The *ardinas* show up at the agency of *O Tempo* in small groups during the day. At the same time there was now a frequent danger, the presence of pirates^{ix} who took the

money from the *ardinas* and this made the staying at the door of the agency much more dangerous for them.

As far as the agency had newspapers to distribute they were given to the *ardinas* who showed up at the agency and in the quantity that they wanted. This meant that no newspapers were kept for those *ardinas* who were in school or that had other activities. The uncertainties about the number of *ardinas* that showed up to take or buy newspapers led to a situation where *O Tempo* was still dependent on them. In fact, the practice of buying the newspaper in a shop was absent in customers and we could even identify a certain resistance to this way of getting the newspaper. On the other side, the number of *ardinas* who assumed a commitment of exclusiveness to *O Tempo* was very low (only 4 out of 32 did have this commitment).

In *O Espaço* the person responsible for printing, organising and distributing the newspapers was Disidori (the same person who was responsible by the distribution of *O Tempo* among the *ardinas* in 1998). At the beginning, all the *ardinas* who sold *O Tempo* in 1998 showed up at *O Espaço* to sell this new newspaper (including all those from S. Martinho). While the two groups of *ardinas* (from Praia and from S. Martinho) were in the practice of selling newspapers, the distribution was decided by Disidori but was executed by himself and by Manu. So, Manu received a certain number of newspapers that he distributed among the *ardinas* from Praia and Disidori distributed the newspapers among the *ardinas* from S. Martinho. In March 1999, the number of *ardinas* from S. Martinho started to decrease and only Manu executed the distribution. However, Disidori decided if the newspapers were distributed all at once or if there were two phases for the distribution. For example, it was common that a number of newspapers were kept for the *ardinas* who were in school and because of that only come to the agency in the afternoon. Once the distribution was finished, Manu came to the Square carrying a certain number of newspapers to give to some *ardina* who was late. He seated himself close to those people who sell goods in the place and there he was during the rest of the day orientating, controlling and in general showing that he was available to receive the final payment just after the selling. The new role of Manu demanded a different approach to the practice itself and to his *ardinas'* peers and, at the same time, a different point of view towards the calculations involved in the situation. All this necessitated a new learning process for him. His previous learning happened (naturally) within his participation as an ordinary *ardina* and was not specially designed for his new role. Participating in different moments of the history of the practice (which organisation was transparent for the *ardinas* about several issues of the controlling process) enabled him to pick up useful elements for that learning. This was the case, for instance, in the table to control the selling process. Now, he was participating in a slightly different practice (the controlling one) in a way that we can classify as peripherally legitimate^x (towards the institution). This enabled him to use resources from what he previously observed in Disidori's behaviour but also from what he experienced himself as an *ardina*. For example, it was possible to observe some similarities with Disidori behaviour in his way of dealing with the *ardinas* but also some differences.

Because there were a greater number of *ardinas*, now they had to circulate more around the city and go further but they kept having their own places for **selling**. It is important to reiterate that just a few *ardinas* followed the rules expressed in December 1998 by the administration of the newspaper *O Tempo* (to avoid selling

both newspapers at the same time). But even those *ardinas* were concentrated close to Manu or in small groups. For example, Manu (with responsibilities in *O Espaço*) was selling *O Tempo* every time he could and Bétu (who was in *O Tempo*) also found a way of selling some newspapers from *O Espaço*. This means that all the *ardinas* were sharing the same places for selling and maintained the relationships among them. They helped each other with the change (exchanging coins and bills), they stayed talking after the end of the selling or in the pauses and they even called for someone who had a certain newspaper if a customer asked for it. Most of the *ardinas* managed to have both newspapers to sell and they arranged this going and getting both newspapers at the agencies or exchanging newspapers among them. In their opinion, the fact that they had the two newspapers for selling was something that facilitated selling both of them. The price of the newspaper for the customer was the same (100 escudos) and the profit per newspaper was 20 escudos in both cases.

The principle accepted by the *ardinas* was that if they took a certain number of newspapers for selling, at the end of selling them they should pay back 80 escudos for each one sold together with the newspapers left. For *O Espaço* the **payment** was carried out in the Square of the city as soon as the *ardina* finished selling his newspapers. There was no collective final moment for this operation. However, because this was the place of selling for most *ardinas* (and the place where most people working at the city were passing by) it was common to see a group of *ardinas* together, some of them selling, some paying and some just observing. For *O Tempo* theoretically the payment was done after selling and at the agency. But we observed that several *ardinas* preferred to make their way to the agency in groups because it was a long way and they were carrying a significant amount of money. So, they stayed at the Square after finishing selling waiting for other *ardinas*. The Square became a place to be naturally together and this gave rise to the same kind of environment that was observed in 1998.

There was in 1999 a quite different way of integration of the newcomers. In *O Tempo*, as was mentioned, the newspapers were distributed directly to all the *ardinas* (except for the three boys protected by Bétu). There was no kind of explicit attitude to a newcomer. But it was possible to identify that a system of integration was in practice. The boys who wanted to start selling newspapers tried to get into this practice through a friend or a neighbour who managed to have a number of newspapers given to the newcomer and assumed a kind of orientation of his first steps. It was also this boy who went to the agency to pay for the newspapers sold by both and not the newcomer. Apparently the system was functioning as previously. However, the price that the old-timer received from the newcomer was different. During the learning phase the newcomer received only 15 escudos for each newspaper sold (instead of 20 escudos). This process was taking place out of the control of the administration of the newspapers but it was a short period because very soon the newcomer got in touch with other *ardinas* and understood the difference in the income. Given that he felt comfortable in the practice he started going to receive directly the newspaper from the agency.

In the case of *O Espaço* it was possible to observe a slightly different process. The newcomers arrived as friends of old-timers but it was Disidori who had the last word about the acceptance and he always tried to have some references about the newcomer. After that, some short indications were given to the newcomer (such as

the price of the newspaper and where to pay) and he started selling. There was no one institutionally responsible for the newcomer but informally the *ardina* who brought the newcomer usually assumed a role in accompanying him. The *ardinas* believed that in this process the most important was the protection and to show the good places for selling. All the other aspects were considered as naturally accessible and easy to learn. But even with differences in 1999 it was clear that some of the fundamental moments of the process of learning to be an *ardina* were lived in the Square both when they were selling together, or paying or listening to the stories of the happenings of the day.

With this description we intend to enable the reader to have a sense of the historical transformations that arrived in the context of the *ardinas'* practice, particularly in what concerns their participation and their learning. Doing so, we are now in position to go deeper in some issues that emerged from our analysis focused on transitions experienced by the youngsters. The main goal of the next section is to support the claim that the *ardinas'* mathematical point of view and their "mathematical" problems are strongly situated in that practice and follow their growing in competence of being an *ardina*.

2. Competence within the practice and mathematics

Julien (1997) argues for a different way of looking for the relation between competence and the taxonomic features. The traditional way of dealing with knowledge (in particular in education) has led us to accept, almost without being contested, that in order to be competent in any domain one first needs to "acquire" a certain amount of objects of knowledge. This idea also seems to be behind the "natural" acceptance of the hierarchical order of the mathematical learning process. It is very often said that in order to be able to deal with (or use) certain mathematical objects or ideas, one needs to learn or understand a lot of other previous "fundamental" mathematical concepts. Only after learning in school a certain amount of pieces of mathematical knowledge the students will be able to apply them in out-of-school situations. However, Julien (1997) points out that competence is not built from facts but in fact it is the other way around, "the basis for the production of facts" (pg. 270) and states that "taxonomic features are not the building blocks of competence; they are a discursive product of such competence" (pg. 270). He builds his approach both from the situated cognition and the connectionist theories.

For the purpose of this chapter, we will focus on some of his ideas trying to enlighten the connections to some particular features of the situated perspective of learning (Lave & Wenger, 1991). We will present and analyse episodes of *ardinas'* life that we believe can give support to the idea that their mathematical knowledge is found in the practice in which they participate. For instance, it is the case of the context, the structuring resources, the habits that gets them through the daily activities in which they are involved and in their discursive practices. We will focus on particular cases, describing some details of the situations but not forgetting their location as moments of the *ardinas'* practice described early on.

The main point is to show how mathematical knowledge emerges from the competence of being an *ardina*. For this purpose we will focus on two instances of

transition: the first one draws on the research situation and not only the *ardinas'* practice, that is the transition experience arises from the situation of the *ardina* acting as an informant to a researcher; the second is one of the practice itself, or in other words, the transition is closely related to what happens within the practice and very much connected with the changes that emerged from the historical transformation of the social and economical world that surround this practice as well as from the integration of new members in the community of practice.

2.1. Calculating within the practice and talking about it

Certain aspects of the *ardinas'* practice require dealing with numbers and calculations. However, as was already mentioned, a great number of *ardinas* (particularly the old-timers) had very low school qualifications and had left school some years ago. Very early on we were faced with the issue of identifying relations between the *ardinas'* participation in their practice and the mathematical knowledge that they developed. During this search several opportunities rose to clarify the relation that *ardinas* established between that knowledge and school mathematical facts. Since the main concern of this research is to gain as much access as possible to the genuine aspects that are typical of the practice in study, we avoided experimental situations and tried to integrate the practice of dialogue (between the observer and the *ardinas*) in what they could accept as true interest in understanding their practice. In Cabo Verde children and youths are easy-going about contact with strangers and usually curious about them too. However, the observer's characteristics - a woman (as were most teachers), speaking Portuguese (the school tongue) more fluently than Creole - might hinder her access to these youths' practice and knowledge. In other words, there was a risk that they might interpret her need to ask questions (many regarding mathematical aspects) more in terms of school evaluation than as a true interest on her behalf in understanding what they knew and she did not.

The dialogues we analysed always took place in the presence of the observer and most of them were brought on by her while talking with the *ardinas* during breaks in their sale or while waiting for the distribution. Within the *ardinas'* practice there were not many moments where the "natural" dialogue between *ardinas* (and with Disidori or Manu) involved reasoning that included discursive elements that are mathematics-related. We figured that this kind of reasoning would happen in payment situations or during the breaks in sales, at which time they verified the money they had. But, in fact, we rarely found this over a period of almost six months for data collection. Usually they checked (the money and newspapers) very discreetly, as if in a private way even though others were physically present. That is, they counted the newspapers and money in silence, without talking about it to each other. It was a painful experience if they happened to find that the money did not correspond to the number of sold newspapers, as if this revealed their inexperience (or lack of skills) regarding the practice. So during these moments they were somewhat reserved in the presence of the others and did not share their doubts.

Similarly, during payment, if any *ardina* faced a situation of disparity between what he expected to pay and what he was asked to pay, his behaviour was discreet and non-argumentative. He would try to delay payment, showing signs of doubt such as looking once again at the calculator screen or recounting the money. This way

Disidori or Manu were forced to do their calculations again, paying more attention to the numbers turning up on the calculator. Some *ardinas* did not even hint at it, simply accepting that the value on the screen of the calculator was correct and handing over the money. At the end they would walk away with their heads low, feeling sad but without saying a word on the subject. If the observer approached them and talked to them about the matter they would often explain what had happened as proof that they had lost money during the sale, or even they had given too much change back, or someone had stolen some newspapers, or else they had not received the correct number of newspapers from the start. Since Disidori's and Manu's calculation processes were completely transparent, they watched the numbers Disidori put in the calculator ("*to see if he's doing the sum well*") and assumed that the calculator never made mistakes and so the result was never defied.

Therefore there were normally few discursive opportunities during this practice as far as aspects that we usually identify as mathematical are concerned, yielding difficult access to these youths' own way of thinking mathematically in their *ardina* practice. However, in some moments this access was possible and we found certain interesting aspects such as their calculation strategies, the supporting elements for these calculations and the difference between thinking and making this thought visible and understandable for others. Situations of conversation with the observer, seen as a person outside the practice but interested in it, also offered the opportunity to approach other parts of the *ardinas'* lives and to talk about their school practice in a natural form, for example.

We are going to look into three aspects we gained access to by means of the analysis of several situations, that we think may help us to reflect upon the relation between the competency the *ardinas* develop during their practice and the mathematics that is present in this practice. First we will discuss the supporting elements for calculations available in their practice which are typical of it and arise in the *ardinas'* calculations processes, secondly the differences between actually calculating or talking about this calculation, and finally the existence of traces of the school discourse and practice.

2.1.1. Supporting elements of the calculation

When we asked the *ardinas* something about the processes they used to make sums while selling, most of them, particularly the newcomers, illustrated their explanation with the act of paying. For example, in 1998 Konka (an 8th grade student) explained how he thought about the money he made in the following manner: "50 newspapers is 5000 escudos, then you take money off to pay Disidori and keep the rest". The value they had to pay for each newspaper (87.5 escudos) was too difficult for them to actually use it in their calculations. However, after a while in the sales business, almost all of them began to handle the value of their gain (12.5 escudos) adequately. The old-timers and some of the more school qualified ones could sometimes explain how they foresaw the value of the money they should pay or make in hypothetical situations that were put before them.

For example, talking with Djeps (one of the most inexperienced *ardinas* in 1998) at several moments during data collection in 1998 it was possible to identify a sequence of different forms.

Epis. 1

One day at the second week in Cabo Verde (while waiting for the distribution with some *ardinas*) the observer asks Djeps if he knows how much he would earn with the selling. He begins to explain spontaneously what should be the gain for 5 newspapers^{xi}.

Djeps — One is 12.5 escudos; 2 is 25;... 4 is 50;... 5 is... 62.5 escudos.

Obs — And about 8 newspapers?

Djeps— (*after some seconds*) 100 escudos.

Obs — How did you think?

Djeps— Two is 25; two more is 50,... are four; plus four are 100 escudos.

Some weeks later he described, for instance, how he calculated the profit on 40 newspapers as follows:

	8	16	32	+8	40	
are	100	200	400	+100	500	escudos

It took a while to get to this point but in 1998 we could find this form of calculating the profit (in action and in their explanations) in several other *ardinas*, whether they went to school or not. In other words, in order to calculate the profit they would often use the corresponding value of 8 newspapers (100 escudos) to support this calculation. Besides one newspaper costing this much, this value was also one of the notes (of Cabo Verde's monetary system) that were most used in selling newspapers. When they went to pay, most of the notes they had were usually 100 escudos and 200 escudos notes, as well as some 50 escudos coins.

Other interesting moments happened in 1998 with Kodé (a competent *ardina* from S. Martinho who finished the 6th grade two years before).

Epis. 2a

Kodé passes near the observer with 12 newspapers and tells her that he already sold 63 (he received 75). She asks him how much did he earn and after some seconds he answers:

Kodé — 887.5 escudos.

Obs — How did you do?

Kodé — 50 plus... 725 plus 13, is 162.5 escudos.

Obs — How did you do for the thirteen?

Kodé — Four ... 50; 8... 100; 12... 150; ... 162.5 escudos.

We can see here that the number of newspapers (63) is split into 50 and 13. (He has added an extra 100 escudos, for the initial 50 newspapers, without realising his mistake.) The number 50 (just like 25 and 75) is frequently used as a base for calculation, since these are the most frequent quantities (in 1998) of newspapers they receive for sale^{xii}. On the other hand, 13 is sub-divided and the values 4 and 8 emerge (50 escudos and 100 escudos, respectively). This addition process which starts with

one of the base numbers of received newspapers is a common case (both in 1998 and in 1999). The case of using the value of the profit of 4 and 8 newspapers is only verified in 1998 ($12.5 \text{ escudos} \times 8 = 100 \text{ escudos}$).

Epis. 2b

One week later, waiting with Kodé for the distribution, the observer presented him with an hypothetical situation: he receives 75 newspapers and after selling them all he will receive 20 more. She asks if he would be able to know the total amount of money earned but using the calculator. He begins to do $75+20$ and multiplies for 87.5 escudos and says "Is to pay". After he divides by 87.5 escudos and in the screen appears 95. He stops and can't do anything else. Then she asked if he is able to do it without the calculator.

Kodé — 1250 is 100, ... less 62.5 escudos, that is 5 [newspapers].

Obs — How do you know that 5 is 62.5 escudos?

Kodé — Twelve and half more 50... as 25 is 2, 50 is 4.

It is now possible to see that Kodé calculates his profit through a process of subtraction from the value of 100 newspapers. This form also arises but only in more experienced *ardinas* or in those with more school qualifications. Indeed, this *ardina* was one of those who had finished the 6th grade but was referred to (by his brother and by Disidori) as having been a very good student and, as he himself said, enjoying mathematics. The difficulty he showed in using the calculator (which is not used at school) shows how this instrument, despite being present in the *ardinas'* everyday life, is actually not part of the artifacts of their practice. We only observed the use of a calculator by some of the old-timers and even then it was just to calculate the value they had to pay. The calculator is an artifact of Disidori and Manu's practice when controlling the *ardinas'* payments, but it is not one of the artifacts of the *ardinas'* practice.

Epis. 2c

Following the talking with Kodé, the observer asks him how much should be the earning for 95 newspapers if they earn 15 escudos for each one. He begins saying he did know.

Obs — You know how much is 100 newspapers?

Kodé — 1500.

Obs — And five?

Kodé — 75... if it is 95 it would be 1425.

Obs — And if it was 20 escudos?

Kodé — Gives 2000 escudos.

Obs — All 100? (*he nods yes*) and the 95?

Kodé — That will give 1900 escudos.

Obs — What is easier, with 15 escudos or with 20 escudos?

Kodé — With 20 escudos

Eu — Why?

Kodé — Cause 5 newspaper gives an exact number, 100 escudos.

It is interesting to look closely at what happens in this dialogue. First a situation is considered, which is hypothetical but involves a rather “uncomfortable” number for the mental calculation that his practice requires. Indeed, it is not easy to quickly reach a multiple of 15 escudos that is associated either with the newspaper’s value or with some of the coins or notes he used. Maybe this is why his first reaction was to deny knowledge of a process for solving this situation. After suggesting the process that he himself used in the previous case, he actually manages to calculate the profit both in the case of 15 escudos and in the case of 20 escudos. One aspect that deserves special attention is the way Kodé justifies the greater facility in calculating with 20 “5... *it gives an exact number, 100*”. This is his way of highlighting the aspect (already mentioned) concerning the search for values that coincide with elements of the monetary system or the price of the newspaper. This was the reason why multiples of 8 were used to deal with the value 12.5 escudos in 1998 and multiples of 5 used in 1999 to deal with 20 escudos.

The calculation of doubles and successive additions are other common aspects that most *ardinas* use in their explanations of different calculations, when the proposed situations involve unexpected or unfamiliar values. For example, the following description is used to explain a calculation with 15 escudos: “2 is 30, 4 – 60, 6 – 90, 8 is...”.

In 1999 it was easier to find *ardinas*, even amidst the newcomers, who very early on could perform calculations both in relation to their profit (20 escudos) and in relation to the value they paid for the newspapers (80 escudos). Also, they more frequently used the reference of 25 (newspapers) than others (50 and 75) particularly among the *ardinas* who began to sell in 1999. At the *O Espaço's* distribution most of the *ardinas* received 25 newspapers; we can understand that those *ardinas* were more familiar with the situation of receiving 25 newspapers than any other kind of number.

In summary we found a habitus on the *ardinas'* practice in dealing with calculations in both periods. That is, all the *ardinas* arrived at using certain elements of the context to support the organisation of their calculations:

- (i) the sequences of mental calculation were structured with supporting elements that concerned values which were strongly related to the elements of the monetary system and to the price of the newspaper;
- (ii) the numbers such as 25, 50 and 75 were used as reference for thinking (the 100 was even more rare in 1999 than in 1998);
- (iii) the calculations began with the values they would have to pay (the newcomers) or the values they made, rarely starting with the money at which they sold the newspapers (100 x x);
- (iv) adding processes were used more often than subtracting processes.

2.1.2. *Internal vs. external conversation or framing vs. re-framing*

On this topic we shall turn our attention to aspects of the practice that gain special visibility in different discourses in situations such as “thinking aloud”, which is typical of the *ardinas*’ practice, or explaining what goes on in the practice to an outsider (the observer). In other words, these are moments when thinking is externalised in a manner that resembles Restivo’s (1998) “thinking as internal conversation”, or that which we considered to be an external conversation and is usually found when the *ardina* explains how he thought. This is almost as if in one of the situations the *ardinas* were more “genuinely” in the framing of their practice and in the other they were in a situation of re-framing of their practice, for outsiders (Goffman, 1991, 1974). This way not only are we able to understand relevant aspects of the practice from the insiders’ point of view, but we also see what *ardinas* use to make it understandable for others.

Two specific examples (Ntóni and Manitu) will be presented and discussed, showing some of the aspects that have been highlighted as supporting elements of calculation.

Epis 3a

During the selling in 1999, Ntóni sits for a moment on a bench (with another *ardina* still selling near by). He puts his bag on the floor and the newspapers he has on his lap. The observer looks at him and understands that he is counting the newspapers he has in the bag. She feels that he has some problem, so she goes to sit near him. He is trying to figure out if he is missing some money or some newspapers.

Obs — Did you lend newspaper to someone? (*he nods no*) No? What then?

He counts again the newspapers within the bag and after the ones he has on the lap. Then he begins counting the money he has (coins and bills).

Obs — How many newspapers do you have there? (*the observer put her hand on the newspapers on his lap*).

Ntóni — 5, 10, twenty (*he knocked on the newspapers with the fist*)

Obs — Do you have 20 newspapers, all together? How many did you take?

Ntóni — 38.

Obs — 38?! Then how much money should you have there?

He recounts the money (the bills), doesn't answer the observer and during the counting looks around as he is thinking.

Obs — So? 38, you have here... 20?

Ntóni — 35 (*assertively*)

Obs — 35 or 38?

He doesn't care about her and starts again playing with the bills and coins.

Ntóni — Twenty (*he is thinking about the nineteen newspapers he carries at the moment and the correspondence to the coin of 100 escudos; after he puts together the 500 escudos bills*)... 25, 30, 35,... missing 3.

This episode shows that Ntóni's actions and words reveal a fundamental concern to make sense of the unexpected situation that arose. That is, to certify whether he has the money and the newspapers unsold corresponding to the 38 newspapers that he thought he had received (and had told so to the observer some time before) or whether he only has 35. His discourse was audible, though not entirely directed to the observer, almost as if he were talking to himself. The structure of the talk shows that he was clearly not worried about what the observer understood. Although he seems to maintain a chain of interaction rituals that would suggest a dialogue (that is, a conversation in which the speech of one projects itself in the speech of the other) it is not, in fact, a true dialogue content-wise. The observer's posture remains that of an outsider trying to make sense of the *ardinas'* actions, behaviours and thinking. Ntóni, on the other hand, now shows that he is not meeting that need of the observer (though he does not refuse it completely). The framing of his interaction is rather that of an internal conversation, of a search for meaning regarding a situation in his practice as an *ardina* where something unexpected has happened (though typical of this practice and in the scope of his skills). This will be actually a problem at the time to pay and brings problems to the satisfaction of his need to participate in this practice.

Let us go into more detail about this episode. In reply to the question about the number of newspapers he has on his lap he first says 5 (that are indeed there), seeming to have heard the question and knowing which newspapers the observer is referring to. However, he then goes on to the number 20 (instead of 19 that are the newspapers he has not sold yet). In these 20 he is already including, besides the 19, the 100 escudos coin he has in his pocket (in the money he counted just a few moments before). When the observer questions him once again, this time about how much money he must have in his pocket in relation to the newspapers he says he has on him, his actions begin by counting the money but the sequence of the words that follow between both of them show that his answers have nothing to do with what he is being asked. In his last utterance Ntóni verbalises a sequence of numbers indicating the counting of newspapers while he handles the money (bills), ending at number 35 and stating that 3 newspapers are really missing. In this check he counts newspapers naming each note to be a given number of newspapers, thus using a counting strategy more than a calculation strategy.

Epis. 3b

Obs — Thirty five, it misses 3 newspapers, so here you have money of 35 newspapers? (*he nods no*) No!... Here you have money of... 500, (*I start counting the money he shows me*)... one thousand and 500, one thousand and 600. One thousand and 600. How many newspapers is this?

Ntóni — 16.

Obs — 16 newspapers and now those more... (*Ntóni count again those he has on his lap*).

Ntóni — 19, here there are 14 (*he points to his bag*) and here are 5 (*he points to his lap*).

Obs — 16 plus 19 how many newspaper is it? Yes, there is 35, there is 3 newspapers missing, how did you lost money? (*he puts the 5 newspapers in the bag and get up*) Do you think that you gave newspapers to another boy? (*he nods no*) Did you count the newspapers when you received them? (*he nods yes*) It was correct?!

Ntóni — Yes (*he carries the bag and gets away from me alone*).

In this part of the episode Ntóni's speeches and actions now seem to be a compromise between the framing of the practice (checking once again) and the talk with the observer (answering her implicit request to gain access to what he is thinking). From this point on he actually answers each one of the observer's questions in a genuine external conversation which is more explicit and which has a sequence of speeches and actions that seems to show some concern in having the other person understand what he thinks (*19, here there's 14 and here there's 5*). Curiously, when the observer counts the money saying "*one thousand 500, one thousand 600. One thousand 600*", she shows she is actually not in the framing of the *ardinas'* practice at Ntóni's level, she does not count newspapers while handling the money, but adopts her usual way of being, which allows her to make sense of the situation more rapidly. On the other hand, we should emphasise Ntóni's final attitude, distancing himself after having given the observer the basic information she requested regarding justification for the happening. In a way it reveals his concern to get back to selling and recover the time lost during this check, but this isolation also shows that he is sad and also a little "ashamed" of not yet being a very competent *ardina* (he is still deceived by others).

Let us now take a look at another episode where we can find what we understand as an example of thinking as internal conversation and where we can go further in the analysis. This one took place with Manitu — a slightly older (age-wise and in terms of sales experience) and more competent *ardina* than Ntóni. Their schooling was very similar, Ntóni still attending the 6th grade and Manitu having finished the same grade in 1998.

Epis 4a

One day in 1999 Manitu received 75 newspapers, he and the observer are talking in the afternoon during some time when he is resting a little. He was walking in her direction, dealing with his money and organising it, and stopped near the bench where she was. It seemed acceptable to talk with him about the checking he said to be usual to do during the selling phase. The observer was trying to understand how he really does that checking...

Obs — And how do you know if the money and newspapers are correct?

Manitu — Count money and count newspapers, then in my head (*he points to his head*)

Obs — So, how many newspapers did you already sell?

Manitu — Sixty three (*as he thinks*)

Obs — 63, how much money do you have there?

Manitu — Hum?

Obs — How much should you have there? For the 63.

Manitu — 800... and 60... 960... I should... 5 *conto*^{xiii} and... seven hundred and...40... so,... 960... no, from 960... and more... 5 *conto* and... 400 no,... 5 *conto* and 40.

It is worth noticing how this *ardina* describes the checking process, fundamentally through counting physical objects (money and newspapers) and then mentally. Next, the observer explicitly refers to the number of newspapers that Manitu has already sold (63) and tries to focus his attention on the money he was handling before (the money from selling these newspapers). From here we can see that she foresaw the existence of a checking process by which the *ardina* would try to verify whether the full money corresponded to the number of newspapers sold. However, Manitu verbalised an unexpected sequence of numbers. He is not explaining how he thought. He is actually thinking out loud, as if this interaction's framing were more one of thinking than of dialoguing with the other person, somewhere very close to the notion of thinking as internal conversation.

Let us go into some detail regarding the sequence he presents so that we understand his line of thought. He starts by thinking not about the value of the newspapers he has already sold (63) but about the value of those he has with him (12 newspapers still unsold) and what he will have to pay for these (12×80 escudos). Then he seems to be trying mentally to calculate the difference between the value to be paid for the newspapers he received ($6000 \text{ escudos} = 75 \times 80 \text{ escudos}$) and these 960 escudos. First he refers to 5 *conto* (if from 6 thousand escudos I subtract a number very close to one thousand naturally I will reach a number close to 5 *conto*) but then he mentions 740 which, amidst hesitations, is changed to 400 and finally into 40. In other words, he seems to be convinced there must be a four in the final value. If his calculating method were purely oral (without the interference of school algorithms) it would be more likely that he would immediately add the extra 40 escudos he had withdrawn when he rounded 960 escudos to 1000 escudos. In that case it would have been quicker to reach the result of 5040 escudos. However, the numbers 740 and 400 allow us to wonder about the possibility of having used another form or of being influenced by certain habits related to the written form of subtracting in school. If he was visualising the algorithm ($6000 - 960$) this confusion concerning the location of the 4 would be acceptable and therefore he would hesitate between the four hundred and the forty.

Although the issue that was raised concerned all the money corresponding to the 63 sold newspapers, the calculations that were performed show he was calculating the value he would have to pay for those 63 newspapers. Let us continue the episode.

Epis 4b

Obs — Five conto and 40... Why? The money for you paying or the money you got from selling for the moment?...

Manitu — No, for paying...

Obs — Ah! For paying.... and for the money all together, paying and your profit...

Manitu — All?

Obs — You still have 12 newspapers... how many did you sell? Is not your profit, is not for paying, what total value of... 63 newspapers?

In this dialogue Manitu confirms his concern in calculating the value he must pay, which leads the observer to think that perhaps her question was not very clear. So she

tries to reformulate it and clarify that she would like him to think about the full money involved in the sale of 63 newspapers.

Epis 4c

Manitu — 63 newspapers? Four... no, 50 is four conto...

Obs — 50 is four conto?

Manitu — Yes... One conto and three hundred,... no 5 conto and three hundred.

Obs — For paying?...

Manitu — No for...

Obs — 50 newspapers is 4 conto? (*he says yes*) For paying... to Anriki?!...

Manitu — Yes for paying, then the profit...

Obs — Yes, then the profit...

It would be interesting to go deeper into this moment of dialogue where he actually seems to be replying to the observer. Yet he goes in a different direction from that shown before. His concern in answering the observer's needs is shown from the beginning of his explanation in the structure of the phrase "50 is four *conto*". As we referred earlier, it is unusual for *ardinas* to talk about their calculations and some of them use a phrase structured like this to clearly express their will to explain what they think instead of simply answering the observer's explicit questions. Now he begins by calculating the 50 newspapers and not the 12 he still has to sell, that is, he now uses a new arrangement of the 63 newspapers (50+13). Although he still thinks first in terms of the value to be paid for the 50 newspapers (4 thousand escudos) he comes up with a value (one thousand three hundred) that corresponds to the sale of the remaining 13 newspapers. This leads to a value that corresponds neither to what he has to pay nor to the value of the 63 newspapers' sale.

In the first part of the episode (his internal conversation) Manitu did not get lost in the sequence of calculations. However in the second (a possible attempt to re-frame his thinking) he confounds his main need as an *ardina* (to pay correctly) with the need arising from interaction with the observer (to explain how he thinks). This interaction involves a different and unusual discourse in this context (that of talking about mathematical facts within the practice), which disturbs his line of thought. This occurs due to this transition between two practices — that of an *ardina* and that of an informant on *ardinas*' mathematical knowledge — that call for different competence. It is possible to say that upon performing typical calculations of his *ardina* practice, Manitu found and used points of support in it that allowed him to keep a solid line of thought. Doing so, he was able to respond adequately to his problem (to check if he has the money to pay for the sold newspapers). Yet when he is faced with the need to speak out this thought he is taken away from his *ardina* practice and it is as if loses those secure points of support, living another practice in which he is not yet confident that he is sufficiently competent. In this practice, not only is the problem defined in a different way, but it can be understood entirely as another problem - not that of an *ardina* in his daily life, but that of an observer trying to understand (and learn with) *ardinas*' practice.

We think that it is really difficult (for an outsider as it is the case of an observer/researcher) to have deep access to the mathematical thinking process

characteristic of the *ardinas'* practice, as the actual language-game of the practice does not include discussions about the mathematical facts we know to be present in that practice. As if although mathematical thinking is present within the practice the mathematical arguing was not "allowed" to happen within that practice. So we almost only have access to the external conversation with which the *ardinas* are re-framing their thinking for the observer (with a different structure from their thinking as internal conversation). Even though the few moments where it was possible to analyse both external and internal conversations showed some key features of *ardinas* thinking about calculations:

- (i) when they have both newspapers and money to check out, they "transform" money into newspapers (500 escudos are 5 newspapers);
- (ii) they were more concerned with checking if they have enough money (80 escudos per each) to pay back the newspapers sold then the total amount of money (100 escudos per each) they should have in their pocket corresponding to the number of newspapers sold;
- (iii) when they need to re-frame their thinking they try to adjust it to what they think are the needs of the observer and doing so they can make more mistakes than when they do their calculations within their practice.

2.1.3. Traces of the school mathematics discourse

Within the vaster project which this paper is part of, we found that all *ardinas* (from the least school qualified to the most qualified as well as those presently at school) ended up using similar calculation strategies, with the same supporting points and scarcely using school mathematics strategies. However, we observed that at certain moments the *ardinas* seemed to use some mathematical knowledge in a school-related form. This was the case of Manitu (in the last episode), for example, but these moments occurred more often in situations born in the interaction with the observer.

At this point we shall analytically describe some episodes that reveal the several forms these "school traces" had.

Epis 5

In 1999, around a tree in the Square where a group of *ardinas* is paying to *O Espaço* newspapers they sold. Ntóni is near by but carrying some newspapers *O Tempo* for selling. He observes the colleagues but he is still paying attention to the movement around him trying to see some customer. The observer is near him.

Obs (to Ntóni) — How many newspapers did you take today?

Ntóni — 38

Obs — Did you take 38? Why?

Ntóni — There were no more.

Obs — There were no more?! Anriki is the one who says how many you should take? (*he nods yes*) Ah! So, now you will sell all of those until the end of the day? (*he nods yes*) And how much is your profit for the 38? Do you know?

Ntóni — Seven hundred and sixty (*slowly, as he was thinking*)

Obs — 760? How did you think? Do it for me.

Ntóni — Multiply (*he keeps alert about what is going around him*)

Obs — How did you think?

Ntóni — In my head...

Obs — But do it, how do you do it in your head, how do you multiply in your head?

Ntóni — At 25 I earn 500; 10 is... 10 is ..., 15 is 300 escudos; 3 is ...60 escudos; 360,... I earn 860.

At that moment another *ardina* arrived to show him something and Ntóni went away with him.

Once more (as we noted in the last point) the *ardina* makes no mistake when he spontaneously performs calculations about the real values at issue. But this does not happen when he has to explain his calculating process to the observer, at a time when he is still concerned with the sale and his attention is turned to this. When he explains how he reached the value of 760 we find again the supporting points that are usual in this practice when: (i) the 25 (the number of newspapers he receives every week at *O Espaço*); (ii) a sequence of additions whose parts are multiples of 5, that is, of 100 escudos. However, there is one slight detail that stands out in this dialogue. Ntóni's first attempt to explain how he reached 760 is done through a word (multiply) that does not belong to the framing of an *ardina*'s practice, but rather to that of school.

As showed in the last episode, in which Ntóni includes words from the school discourse in the dialogue with the observer, this often happened with other *ardinas*, even with those for whom school was more of a memory than anything else. When trying to explain to the observer the procedure for calculation (and particularly those who had more difficulty in expressing themselves in Portuguese), sometimes the *ardinas* used words that seemed to show some relation to their thinking, although these words were not always used correctly. The following episode illustrates this.

Epis. 6

The observer is talking, in 1998, with Pitchiu (an *ardina* from S. Martinho who left school 4 years ago) before the distribution. She is with a colleague from Cabo Verde (Ana) who acted during the 3 first weeks as an interpreter between the observer and some *ardinas* who have more difficulty understanding and talking Portuguese. They were talking about the profit they do from selling and Pitchiu is saying that he usually sells 100 newspapers so he has a profit of 1250 escudos. She asked how much he would earn if the profit for each newspaper was 15 escudos. He needed some seconds of concentration but he said with no doubts that it would be 1500 escudos. The observer asked him to explain how he had thought. During several attempts he was doing, in creole, to the observer or to Ana some dialogues developed such as the following.

Ana — Think aloud, make the calculation aloud...

Pitchiu — I do in my head...

Ana — How much is it, your profit?

Pitchiu — One thousand and five hundred.

- Ana — How did you do it?
 Pitchiu — In head... I added, added...
 Obs — How did you add?
 Pitchiu — Added 15 times 50,... it gives,... gives... gives.... 750 escudos times 750 escudos...
 Kodé (*an ardina, friend of him that was near them*) — No, he has done the calculation of 50 times 15.
 Pitchiu — 50 times 15 gives 750... 750 plus 750 gives 1500
 Obs — Ah! And 750 how did you think?
 Pitchiu — Added from 1 to 50... 50 is half of 100. Added 15 times 50.

Here the verb “to add” seems to be used more to adapt to the observer’s discourse; it is not a word of the *ardinas*’ practice, there the words they normally use are “plus” and “times”^{xiv}. We may think that their difficulty simply stems from their foul Portuguese, but in other situations, where they explained other aspects of their life as *ardina*, they commonly turned to Creole to make themselves clear. They had already realized that the observer understood them well in Creole (actually one of the *ardinas* with whom she most talked almost always spoke in Creole). This led us to consider that talking about the calculation processes they use is not a part of the practice. This result is coherent with the observation and with the information gathered from several elements (*ardinas*, Disidori, Manu, former *ardinas*).

But the presence of the school influence was felt in more than scattered words in certain moments. We can see this in the following dialogue. It takes place in the Square, between Diku (in 1998) and the observer, during a break in the sales. This *ardina* was from S. Martinho and he had attended school until the previous year, having been a very good student and concluded the 6th grade.

Epis. 7

The observer is talking with Diku about the selling of the day before when he sold 35 newspapers.

- Obs — How much was your profit?
 Diku — Twenty five is 325 escudos plus 10 is 125 escudos.
 Obs — When you were young did you think like that for your calculations?
 Diku — No.
 Obs — How was it, then?
 Diku — I did with the calculator.
 Obs — Did you learn with somebody?
 Diku — No, was by myself...
 Obs — How did you begin to think about it, did you remember?
 Diku — 25 at 12.5 escudos gives 312.5 escudos;... 50... 25 is half of 50,... gives 312.5 escudos ... 50 gives 625 escudos.
 Obs — Did you already know that 8 is...?
 Diku — 100 escudos
 Obs — How did you begin to think about 8 newspaper? You never take 8 newspaper, do you? (*he nods no*)

Diku — 8 is 100 escudos because 2 gives,... one is 12.5 escudos, 4 I'll earn 50 escudos and 4 is half of 8, 8 is 100 escudos, 16 is 200 escudos, 20 is 250 escudos.

Obs — 20? Why did you jump from 16 to 20?

Diku — Because 16 is 200 escudos, 4 more is 50 escudos, so 20 is 250 escudos.

In this episode Diku clearly explains his calculations by means of the same supporting points of the practice that we referred to earlier on (the 8, the 25 and 50 and the doubles). However, the structure of his explanation is somewhat different from what we usually find in the dialogue with *ardinas* who are less school qualified or attended school longer ago. When the observer asks the first question his answer is not the final value but the way of reaching it. Other *ardinas* would normally reply the former, making it necessary for the observer to make explicit the question of how they did it so that they would describe processes. It seems that this particular *ardina* acknowledged a sort of questioning that is typical of the school dialogue (from a teacher to a pupil). In this sense, he understood that the observer's question is not because she can not manage to calculate the profit but because she is interested in the processes they use to think.

Further on, the observer does not give him a specific situation to think about. Under these circumstances with other *ardinas* it was necessary to reformulate the question so as to offer a situation in which they could place themselves. Diku does not need that and organises his explanation around values (50 newspapers and then 20 newspapers) which he feels are appropriate in order to satisfy the observer's curiosity. As a good informant, who is aware of the observer's implicit interests in the questions she asks him, he also demonstrates greater ease than most *ardinas* in explaining processes explicitly. In other words, he shows himself to be more at ease in the situation of talking about the mathematical aspects of his practice than we had come to expect from interaction with the other *ardinas*.

Another distinct form in his discourse is shown in the phrase “25 at 12.5 escudos is 312.5 escudos” in which he uses a formulation that is not part of the other *ardinas*' usual patterns of dialogue, even with the observer. This form of discourse seems to be quite close to the usual language-game in school mathematics (form of talking in the school mathematics discourse), when relating a variable with its unit values in the way we have just seen. Diku uses the typical way other *ardinas* have for explaining this type of relation “one is 12.5 escudos, 4 I'll earn 50 escudos”. On the other hand, he often tries to highlight the relations between quantities (25 is half of 50) in order to justify the calculation with the corresponding value in money, thus demonstrating a kind of pedagogic attitude. But at the same time, when he was asked why he went from 16 to 20 he calls to a logic quite specific to the *ardinas*' practice: the value of 4 newspapers (50 escudos) that corresponds to a widely used coin. On the other hand, the numbers he used to explain the calculation he made to reach the profit for the 35 newspapers are also typical of this practice (the 25 in particular). So Diku seems to show that he is aware and uses the strategies typical of the *ardinas*' practice but he is also sensitive to the observer's interest in an idea of mathematics as something that has to do with relations between numbers and not only with calculation methods.

Let us now look at another episode where the same *ardina* demonstrates forms of thinking that are equally different from most *ardinas*, but in which we can identify certain characteristics that reveal not only aspects of the practice but possibly also the school framing.

Epis. 8

In 1998 two *ardinas* (Djeps and Diku) are seated outside *O Tempo* agency waiting for the distribution. The observer is seated with them and they talk about the possibility of a profit in each newspaper being 15 escudos. She asks how much could they earn for 10 newspapers in that case. Djeps explains to her by thinking in groups of two newspapers and says "2 is 15 escudos, 4 is 30 escudos...". He takes a longer time to say the value of 8, so Diku decided to intervene saying that 10 is 150 escudos. So she asks him how he thinks about it.

Diku — 15,... as 10 newspapers is,... at 12.5 escudos is 125 and it rises more 25... is 150 (*he makes a movement with his hands to show the idea of rising*).

Obs — How did you think that it rises more 25 escudos?

Diku — I did... each newspaper, 2.5 escudos for one, plus 2.5 escudos for another,... and so (*he indicates with the fingers one by one*)

Obs — Did you add one by one?

Diku — No! I add all together... (*he makes a movement with his hands to show that is referring to the totality*)

Obs — Everything? All at once? How did you add all at once?

Diku — 10,... is 150.

Obs — And if it is 20 escudos for each newspaper? How is it for 10 newspaper?

Diku — 200 escudos.

Obs — Why?

Djeps— 'Cause 5, if it is 20 escudos each...

Diku — 5 is 100 escudos...

The distribution begins and they go to receive the newspapers.

The observer was surprised by Diku's explanation for it was completely different from what happened with other *ardinas* (who used Djeps's strategy). His way of thinking led him to link both situations, the current one and the hypothetical one, relating the current unit price with the hypothetical price that he was faced with so as to use a value known to his practice (125 escudos) to calculate what he would make faster. Curiously, when he explains in more detail how he reached 25 escudos he clarifies the difference he found between the two unit values (2.5 escudos). However, he describes the global procedure as an adding process (just like Djeps), both in words (added) and in gesture (counting one by one with his fingers). Next, with the observer's question he realises that he gave the wrong idea and tries to explain again that he thought in whole terms. In other words, he may be referring to a multiplication without, however, naming this operation adequately. He might have done so by using the word "multiplication" (a word from the school frame) or, as Kodé did in episode 6 with Pitchiu, by using the expression "times" (a practice word). It seems that for Diku

the 2.5 escudos x 10 was immediately visualised as 25 escudos, almost as if there were a “blind conscientiousness”^{xv} (Bloor, 1997) of what happens in the multiplication by 10, but also knowing that this change results, in fact, from the successive addition of equal numbers.

With these two episodes we intended to highlight how this *ardina* makes both “worlds” of the situations lived with the observer — the *ardinas'* practice and the school practice — visible through small dialogues and how he lives this transition entering a new role — an informant on *ardinas'* practice. This is clearly suggested by the situation that is not common in this practice (dialoguing with an outsider of the practice who has certain features that resemble those of a teacher). We must note that this *ardina* talked (in quite good Portuguese attending to the fact that he was out of school for almost one year at that time) with the observer several times and spontaneously about school, reading and mathematics at school, things that he said he liked very much. Besides that his motives to participate in this practice were somewhat different from the others. He liked coming into the “big city” watching people and the movement of the city, to participate in a different life from what was possible in his very small village. To participate in the sale satisfied this willing in a way that was acceptable to his family. All this contributed also to his explicit preference for staying with the observer – chatting with her in the Square rather than going and selling more. We can speculate that his particular empathy with the observer drew also from seeing her as closely related with the other “world” he wanted to participate in – a scholarly, literate and urban world. Offering himself as a good informant was one way of positioning himself in a transition experience: participating in an alien practice to him (the research practice) about a practice where he was a full participant (that allowed him to show and explain what he knew about it) and that the observer was trying to know better. But participating in this experience fulfilled his own needs and not only the researcher’s (at the same time that he was aware of being helpful to her). In this sense it seems to us that the way the participant understands the relevance of the transition experience to his own life, to fulfil his own purposes or needs, can be an important element for how he will involve himself and take the best of it.

2.2. Living in between

One of the main features of the *ardinas'* practice is its dynamic nature that demands adaptation to new interactions and new rules that arise from the historical transformation of the context, both from the integration of new members (a “natural” movement within the practice) and from the changes pushed by some instances of the wider socio-economical context in which the practice is part of (that is, from the movement “outside” the practice). Another aspect that seems to play an important role in the constitution of *ardinas'* practice is the fact that their activity is lived between an institution (trying to adjust to a market-driven modern economy) and the daily life of ordinary people (still ruled by a very traditional and communitarian way of living together). Those two features of the practice turn out to be very important in bringing particular demands to the *ardinas'* daily life, constantly confronting them with different values. The dynamic nature of the practice together with the constant

living in between two socio-economical systems seems to become another instance of the transition experienced by the *ardinas* during participation in the practice.

We will now focus on the ways participation is sustained within the practice and how mathematical facts gain sense from the development of competence in the process of increasing participation from newcomer to old-timer.

2.2.1. Sustaining the participation and the practice

In the *ardinas'* practice it was particularly interesting to observe two forms of sustaining the participation in this practice. On one hand, according to the way Lave and Wenger (1991) refer to the "sustained participation of newcomers, becoming old-timers" (pg. 57), the *ardina* can be viewed as an element of a community of practice acting in order to maintain the possibility of the existence of the community. On the other hand, when the *ardina* reformulates the institutional "rule" in order to be able to maintain his participation in the practice, this seems to us to be very close to Wenger's (1998) call of attention to the "non-participation developed with respect to the institution (as) an integral part of their identities of participation in their own communities of practice" (pg. 172). We shall now turn to these two aspects of sustaining the *ardinas'* practice.

In the brief introduction to the *ardinas'* practice (section 1.2.) we described what we call the "learning curriculum" (in the sense of Lave & Wenger, 1991)^{xvi} which existed in 1998 (with quite a lot of input from Disidori) and in 1999 (without any institutional interference). In short we may say that over these two years this curriculum had a similar form as far as the organisation of activities for newcomers was concerned (which allowed for a safe participation, both for them and the institution itself). This organisation was perfectly integrated in the vaster organisation of this practice's activities and apparently it seemed no different than other *ardinas'* normal practice. However, if we look carefully we can find certain nuances that may be relevant. For example, the fact that newcomers received newspapers from an old-timer was a sign of their participation being peripheral, albeit legitimate. In the same way, the smaller numbers of newspapers which newcomers handled at the beginning made their task less demanding both in physical terms, in keeping the newspapers in good condition, and in the level of attention that is necessary to deal with the money involved. Besides, limiting the sale of those who are learning to the Square or close to an old-timer, reduces the difficulty of these *ardinas'* tasks. Doing so, they need not pay attention to a multitude of factors involved in the practice, such as the time and place (whether it has movement or not), the pirates that might be hanging around them, the ability to prevent future situations and thus organise the change to give the client so as to have enough coins and bills for other situations. Only after some time with an old-timer do newcomers venture further and receive more newspapers. This evolution in the sense of a gradual autonomy from his partner makes the newcomer's activity more complex, requiring greater knowledge of the city and its rhythms, for example. Apart from this, it introduced greater possibility of conflicts with *piratas* or situations where the *ardina* might give the wrong change and even lose money. The consequences of many of these situations would only be felt during payment, hence the awareness of the need to frequently check the correspondence between the newspapers sold and the money in their pockets. In general, stories about this kind of

issue were told right at the start, but the newcomer would only gain the habit of actually checking much later in their learning process.

In 1998, deciding which old-timer would be responsible for a newcomer was Disidori's decision but this was well accepted by all the *ardinas* ("I like to help people" they would say) and it was even a way of acknowledging their status inside the community. In 1999 the social organisation of the *ardinas*' practice had to adapt to the birth of another newspaper (*O Espaço*). In the initial description we spoke about the main changes that occurred but it is now important to point out certain aspects, particularly in relation to the newcomers' integration. While attempting to keep up a learning system for the newcomers (a part of the history of the practice) by themselves, the old-timers would adapt it to the current situation and show how they felt regarding the newcomers' integration. The entry of newcomers increased the number of *ardinas* selling, which could mean that each one would earn less money. When the old-timers took on the role of helping the newcomers, for a time they placed themselves in a relatively powerful position as gatekeepers of the community. This power helped them to increase their gains by establishing a price for their "help" and simultaneously to control a situation that had the potential to threaten their own place. The internal acceptance of this situation also shows how the importance of a privileged relation between an old-timer and some newcomers is acknowledged (both from old-timers and newcomers), at least in the beginning. The newcomers face a new and sometimes frightening situation (in the relation with the *piratas*, in handling a lot of money and in the responsibility this involves) and have the comfort of a "friend's" protection and help. Actually they usually referred to a need for protection rather than for learning. As we noted before, the learning situation was not limited to the privileged relation between an old-timer and a newcomer; mutual support between peers (other newcomers or else slightly older with little experience) was important also in sustaining the newcomers' participation. This adjustment in the learning system not only reveals the dynamic character of the practice but also the agency of *ardinas* as a community (before the institution) in which certain elements have the ability to take on a power^{xvii}.

Sustaining the participation has another important aspect which emerges from confrontation between the *ardinas*' need to maintain the possibility of making money with some regularity every week and the rules defined by the institutions with which they interact in their practice. For example, some of those who intended to sell in certain spaces (public services, banks,...) took the care to dress and behave in a particular manner. However, they all realised that somehow their participation had to take place within the social organisation of the practice, particularly regarding the norms of the agencies whose newspapers they sold. When we tried to identify the fundamental lines of this organisation from the *ardinas*' point of view, we came face to face with the strong presence of the idea of "rule", both in their discourse and in their behaviour. We realised just how strong these rules were and how they sometimes sustained the *ardinas*' participation. That is, when these rules were made visible and we understood their inclusion in the *ardinas*' activity (considered to be rules in the sense that they help also to constitute the practice) and how they interacted with the organisation imposed by the institution. Some of these rules were very explicit and institutional (in the case of *O Tempo* in 1999, or the obligation to pay for the sold newspapers), others less, arising at times from interactions between *ardinas*. Although their most natural attitude was to accept and respect the enforced

norms and hierarchies, sometimes they needed to "re-read" these norms or adjust their procedures so that they would appear to follow them. It was a way of the *ardinas* themselves managing some of their needs in the strictness that was imposed to the *ardinas*' group. For example, how they modified (in action) the rule of *O Tempo* "not to sell *O Espaço* when selling *O Tempo*" transforming it into "not to make visible *O Espaço* newspapers when going to get *O Tempo*" made it possible for them to sell both newspapers at the same time, increasing their ability to make money but also to sell either of them faster. Therefore, even in the case of those rules that were defined by the Institution, the *ardinas* would adjust them, on the one hand, for sustaining their participation in this activity but, on the other, for sustaining the practice that should continue for them to satisfy their main goal. At the time, without a set of *ardinas* who would constitute a community of practice (guaranteeing the newcomers' learning) it was in fact not possible to sell newspapers in Praia. So it became a mutual need (for the *ardinas* and for the newspaper's administration) to maintain this practice and through it to sustain the participation of *ardinas* in the practice. It was possible to notice that some *ardinas* were selling both newspapers at the same time. The ones who bought *O Tempo* (and not take) maintained *O Espaço* visibly when they went to *O Tempo* board to receive their newspapers. However nobody there argued about that situation with them. Things were left in the realm of the unsaid, of what is not convenient to clarify completely since it involves risks for the institution itself. Situations of disruption did no good to either of the sides so many situations did not lead to a point of total rupture, whether by the newspaper (Disidori, Anriki, newspaper board, Manu) or by the *ardinas*.

As Wenger (1998) suggests, "such a concept of practice includes both the explicit and the tacit. It includes what is said and what is left unsaid" (pg. 47). It seems as if in periods of main changes occurring within a practice the need for sustaining both the participation (and thus the community of practice) and the practice emerged more strongly than in moments where the changes evolved more naturally from the internal evolution of the system (as it was in the case of the daily integration of newcomers).

2.2.2. 20 is not the same as 100-80

As we mentioned before, in March 1999 there were, at the same time, two different forms of *ardinas*' access to the selling process of *O Tempo* ("taking" and "buying") but all the *ardinas* took newspapers in *O Espaço*. Some *ardinas* experienced both and they lived those two systems when they sold both newspapers. Within the *taking* system, *ardinas* were allowed to receive a certain amount of newspapers, to give back to the agency the newspapers they were not able to sell and to pay for only the sold newspapers at the end of the selling process. However, some *ardinas* were obliged to buy the newspapers they received if they wanted to participate in the selling (as was the case of some old-timers from Praia). From the start all *ardinas* were on the regime of *taking*; however, when one of them did not pay in time (thus showing they were not serious), *O Tempo* changed the system for those particular boys. Usually, two or three weeks after, these *ardinas* would come back to the agency asking to be allowed to sell again. If they were accepted, usually they were integrated in the new system (buying). In either system, the money earned from each newspaper was the same (20 escudos). But when the *ardinas* took they had to go back to the newspaper agency to

return the unsold newspapers and pay for the ones they sold, but if they bought they should pay 80 escudos for each of them as soon as they received them.

Let us now see how the *ardinas* viewed this situation. We begin presenting 3 episodes, which will be discussed jointly, despite having occurred at three different moments. They involve four *ardinas* who had been serious regarding payment procedures with *O Tempo* up to that point. They sold newspapers on a taking regime and were attending the school in the 6th grade. Ntóni was the youngest in the sale, Djoka and Lulu were also young but had more experience than Ntóni, and Manitu had already begun to sell in 1998. We shall also comment on the opinions of certain old-timers about the same issue. The dialogues happened with the observer and dealt with the question of which system they considered the best. Therefore, the problem was not expressed with the intention of assessing whether these *ardinas* thought the profit of 20 escudos in both systems was identical. Because the profit was the same in both systems (20 escudos) what we intended was to understand just how relevant this mathematical fact ($20 = 100 - 80$) was considered to use as a base for their choice between the two systems.

Epis. 9

Ntóni and Djoka are discussing which newspaper is better to sell and what is the best access way to sell (buying or taking). The observer is the one who provokes this subject. They disagree about it, Ntóni saying that buying is best and justifying like this:

Ntóni — When we take newspaper we can't take off our money...

Obs — Don't you, the profit? When you take, how much is the profit?

Ntóni — 20 each.

Obs — It's 20 each newspaper? And when you buy them?

Ntóni — Pay 80 and sell 100.

Obs — 100, and the profit? (*Djoka says that is 20*) It's also 20? Is it the same thing? (*Ntóni stands still but Djoka nods yes*) But,... why is it best to buy?

Djoka — (*makes a movement with the hand towards his body*) Taking.

Obs — Is it best to take?...

Djoka — If you don't have money...

Epis. 10

The dialogue is between the observer and Lulu, in the Square, after finishing the payment of *O Espaço* he sold. They are both seated on a bench, together with Manu and Ntóni that observe them without speaking.

Obs — (*to Lulu*) Did you take or buy them?

Lulu — Take.

Obs — Take, hum... and what thing is best, to take or to buy?

Lulu — Buy.

Obs — Buy is best? Why?

Lulu — 'Cause is yours... (*he points to himself*)

Obs — But do you earn the same? (*he nods no*) No?! Do you earn more? (*he nods yes*) How much? When you take how much is it?
 Lulu — When I take earn 20 in 100, to buy is... 30...?

He hesitates a little and look to the direction of Ntóni and Manu as if he is asking for some sign of agreement. Ntóni responds to his demand saying that it is the same.

These two episodes allow us to reflect upon two aspects: (i) the choice made by two of the *ardinas* and the way they begin to justify this choice; (ii) the apparent difficulty in clearly expressing the real profit in each regime. Ntóni and Lulu perceive the “buying” system as having more advantages and express essentially sociocultural (and not mathematical) reasons for this. For example, their first justification seems to highlight a concern for a sense of possession (“*you can’t take our money*” and “*it’s ours*”). On the other hand, both show doubts when clarifying the profit in each of the regimes. Ntóni is able to explain the profit correctly when buying by using a formulation that could indicate an ability to understand that it is, indeed, 20 escudos. Yet quite unexpectedly he reveals difficulty in realising that the profit made in both regimes is one and the same. Lulu, on the other hand, does not seem to know how much is earned in the buying regime. We can interpret these hesitations in two ways. Since neither of them has experienced the buying regime they do not yet know the complexity involved “from the inside”. In fact, for the *ardina* the taking regime is less risky, for it guarantees an effective profit of 20 escudos for each sold newspaper, unlike the other regime where this profit is smaller if the newspapers are not all sold. This also seems to reveal that these aspects are not objects of conversation between the *ardinas*, they must live the experience in order to actually “learn” which variables are involved and the importance each one must have for making decisions. But it is also possible to think that the fact that the sense of possession is more evident in the buying system may be relevant and lead to a situation where the “objectivity” of $20=100-80$ (a mathematical fact) is meaningless for them in the light of social references.

The third *ardina* involved in these dialogues (Djoka) raises another element (“*if you don’t have money*”) in his justification, which we also found in other *ardinas* (who were more experienced than Ntóni and Lulu). This seems to support our previous interpretation. It is the case in the following episode.

Epis. 11

Later in the same day, the observer has a long conversation with Manitu as he sits near her in order to rest a little and showing interest in talking with her. They talk about several aspects of his *ardina* life but also about his family and the school. The observer felt that it could be an opportunity to talk with him about the two systems (taking and buying).

Obs — What thing is best? Take or buy newspaper?
 Manitu — Take, for instance,... if one time you don't have money to buy you stay as,... you can't sell (*he says this last idea in a slow manner as if he was showing sadness*).

Both Manitu and Djoka justify their decision for “taking” as the best regime for those who do not have money, relating it to the access to selling. They seem to suggest that this situation is seen as a sign of inferiority and dependence. In this case they have to take advantage of the organisation that is institutionally imposed so as to guarantee their participation and, therefore, satisfy the need that led them to participate in this practice in the first place. On the contrary, in the buying regime there is a possession of something from the start. When buying the newspapers at the agency, the *ardina* has a reasonable amount of money from the start that he uses to exchange for something (the newspapers) and from that moment on all the money he makes will be his. In turn, the *ardina* who takes newspapers receives them without giving anything in exchange. At the end of the sale of two *ardinas* in different regimes, the way each of them faces the others (and it is seen by the others) is really different. For the *ardina* who takes, the act of paying is a moment where he delivers a great amount of money, leaving a very small part for himself. On the contrary, the ultimate image of the buying *ardina* is of possessing a great deal of money, all he got during the sale.

Curiously, there never occurred to the *ardinas* an opposite (but possible) interpretation. In fact, when the institution delivers newspapers in exchange for nothing, they do not see this as an attitude of trust towards the *ardina*, as an acknowledgement that the person is trustworthy. *O Tempo* introduced this regime in two types of situations. At the beginning it was used to distinguish between the Praia *ardinas* and those from S. Martinho (as a punishment for these for having a privileged relationship with Disidori with whom the administration was on bad terms). Later it was adopted as a way of dealing with those who did not show up at the end of the sale to pay. The latter is an attempt to reduce the risks surrounding the delivery of newspapers to youths coming from families with economic difficulties without having to control them more directly (in 1998, done by Disidori). This attitude of *O Tempo* was interpreted by the old-timers as a sign of less solidarity and of competition, both between groups (S. Martinho and Praia) and between newspapers. On the other hand, they realised (some of the old-timers said it clearly to the observer) that it was a way the newspaper could get the same service for a smaller cost for it did not need to pay an employee to perform Disidori's role. The *ardinas* felt that it is better to take than to buy, since, for example, "when you don't sell everything the agency recovers whatever is left" or (regarding the buying regime) "you've to be careful and not ask for many 'cause you won't sell them". Besides this, they clearly say that "this year *O Tempo* doesn't trust the *ardinas*" and present this as one of the justifications for not complying with their obligation to pay. In 1999, the *ardinas*' daily practice runs amidst two systems whose organisations reveal the presence of values which, in one case, have a more traditional character and, in the other, are close to those of a mercantile economy. The way the *ardinas* behave with each of the newspapers^{xviii} shows, in several moments, how they perceive this dissimilarity, how they interact with it and how they organize themselves in order to satisfy the needs underlying their integration in this practice.

At several moments of the two periods of data collection, when asked to characterise what they thought was "a good *ardina*" all of them started by saying that "he should follow the rules" or "be honest" referring most of the times to the payment rules. After that, they accepted that he should be able to sell quickly, know the best places, address the client correctly and finally give change. Besides what was actually said by the *ardinas* when questioned, most of them also had certain behaviours that revealed a

great concern in being correct about payment procedures. The act of paying the sold newspapers structured their way of living during the whole sale time (as was described in 1.2). On one hand, it was during these moments that more collective situations were experienced, where behaviours came out and stories of the day were shared. Although largely lived on the streets, these were the most structured moments of their interactions with Disidori and Manu, and the procedures were more explicit. In fact, these were the only moments when cultural artifacts of the practice were used in a consistent way – the calculator and written records on which *ardinas* were selling on that day and how many newspapers they had taken. Besides, this was when large sums of money were visible (the money is in fact the first appeal that leads *ardinas* to this practice). These circumstances (artifacts and conditions of the activity) helped confer a certain solemnity and respectability to the moment, that is, they helped define the status of this phase in relation to the others (distribution and sale) and were genuine structuring resources (Lave, 1988)^{xix} of their selling activity. The newcomers, in an increasing participation (where the peripherality was legitimate so it was positive), also have access to this “world” of payment, learning to recognise its importance. In 1999, the visibility of payment procedures at *O Tempo* as a component of a community’s practice, was more diluted. Requiring a new trip to the agency after satisfying the first goal of the *ardinas*’ activity (making money from selling newspapers), as well as the fact that there is one group which does not have to do this (mostly the old-timers) also hinders the wish to perform well. Starting with what we could refer to as a mathematical fact that the *ardinas* knew (20 as a result of the subtraction 100-80), it is interesting to see that they actually even refer to a lower profit in one of the cases, based on essentially sociocultural justifications. They become fully competent in the practice when their participation in it is a “full participation” (Lave & Wenger, 1991, pg. 37). In that process of increasing participation they are, for instance, using the resources that structure their activity, experiencing various situations which arise from the practice’s organisation, therefore understanding the diversity of relations involved in it. Only then do they compare the two systems of access to the practice with more “objective” references, which raise the fact that the profit might not be exactly the same in the two situations. In other words, the competency of being an *ardina* is the starting point for building bases for the knowledge (or re-contextualisation) of facts, in this case mathematical facts.

It seems that the apparent less visibility of control on the part of the agency (*O Tempo*) made stronger the need for cohesion among the *ardinas*’ group. This led them, for instance, to re-write the institutional rules in order to avoid colliding with the institution but making it possible to keep a certain sense of belonging to a community. Besides that, the *ardinas*’ greater autonomy (also apparent in not being pointed out by the authority - any old-timer could help the newcomer) made clear the need they felt for a certain learning/protection system that helps the newcomer but keeps the control within the gatekeepers of the community (the old-timers, protecting the individual *ardina* but also the community, sustaining both the participation and the practice). This situation came in a period of great instability within the practice as a result of the co-existence of two different sets of rules (in the two agencies) and, in one of the agency, two different systems (buying and taking), together with a more visible effort from this agency to move forward to the market-driven economy emerging in the country. That is, the *ardinas* were pushed to live frequent transition experiences in a practice that is no longer ruled only by the agencies or by one

system, but where they all together constructed their own rules, the context of the practice, as if they were living their daily activity in a kind of transitional space. It was possible to observe different ways of dealing with those transition experiences particularly between the newcomers and the old-timers and we could realise how they interrelate, for instance, with their perceptions of facts that apparently are objective (as it was the case of the amount of profit). On the other hand, the contribution that the participants recognise in their participation to their own life projects seems to constitute an element that plays a role in the way they engage on the transition experiences and how they bring in the resources available not only in the practice where they are situated but also from others practices where they had participated.

3. What did we learn?

A powerful idea that we would like to bring to the fore again is the understanding of “learning as an integral part of generative social practice in the lived-in world” (Lave & Wenger, 1991, p.35). One should recognise that this means that participation in social practices does not merely influence otherwise autonomous psychological processes. Under this perspective, learning means changes in the ways that a person participates in social practices. So, it seems to us that if we propose ourselves to understand how mathematics learning connect to social practices where mathematics' use is present the social practice is the "primary, generative phenomenon and learning is one of its characteristics” (Lave & Wenger, 1991, p.34).

Our main concern in the whole research project was driven by the need to better understand the relation between social practice and (mathematics) learning. With this main goal we decided to observe, describe and analyse a social practice where we (and the participants) identify the use of mathematics (even with different meanings). In this work we need to situate that social practice within the social world where usually it takes place but also to have some elements that allow us to analyse how the participants evolve through the historical transformation of the practice. Through the work already done in the main project we confronted ourselves with some issues that emerged from the analysis of particular moments of the *ardinas*' daily practice – those we understood as experiences of transition that took place, some within the historical transformation of the context of that practice and others because of the research situation. Focusing the analysis on those moments appears to be fruitful in bringing to the front some aspects such as: (i) the relation between the boys' participation in the *ardinas*' practice, their learning and use of mathematics in that practice; (ii) the connection between the mathematics of the practice and the school mathematics and its presence (or absence) within the *ardinas*' practice.

From the very beginning we stressed the analytical tools that enabled us to go deeper in our research (some of them may not be very visible in this chapter although they played a role to the preparation of some discussions). For instance, in order to describe and analyse the presence, meaning and importance of rules in the *ardinas*' practice we were relying on Wittgenstein (1992/1953) perspective about rule-following and about meaning as grounded in collective use, as well as on Goffman's analytical framework of rules (particularly on what he called regulative and constitutive rules). The study of Goffman's social theory went in parallel with our reflection on research questions and data. And this led us to find as relevant

Goffman's (1991/1974) rationale on frame analysis. This second aspect of his approach to the study of interaction order played an important role in our use of Restivo's idea of thinking as internal conversation (different from external conversation) as one way of making sense of differences identified in the *ardinas'* discourse in some particular moments. Our interest on the approaches of Restivo (1998) to thinking and Julien (1997) to competence came from our effort to understand how *ardinas'* mathematics knowledge emerged from their participation in the practice.

Along with our concern with those issues about mathematics learning, several methodological issues turned out to become relevant and part of our reflection on our research practice. The type of research questions together with the theoretical and analytical approach we took, suggests that we present what we have learned from our own participation in the study mainly around two aspects: (i) what it concerns with regard to the perspective of learning as participation in a social practice; (ii) what relates to the methodological aspects of the research.

3.1. About learning as participation in a social practice

With the analysis reported we intended to give evidence of changes in the forms of participation of the *ardinas* in a practice and therefore to show instances of learning. We will now summarise our findings mainly in two aspects that appear to be the most fruitful ones to contribute for expanding our understanding of mathematics learning as (legitimate peripheral) participation in a social practice: (i) what emerges when we look at *ardinas* practice as a social practice; (ii) what comes when we look at *ardinas* as individual participants in that practice.

3.1.1. Ardinás' practice as a social practice

The analysis showed evidence of the **dynamic** character of *ardinas'* practice that came from changes that took place both around and within the practice, and that play a role in the way participants participate in it. On one hand, the arena as a pre-existent entity independent of the participants has its own evolution according to forces and relations that emerge from its nature (an economical institution) and position within the society as a whole. It was the case, for instance, of the existence (in 1999) of two newspapers instead of only one (in 1998), or the change of the person who used to control the *ardinas'* work. On the other hand, as the *ardinas* only had an informal link to the agencies and most of them didn't integrate their participation in this practice within a professional perspective (it was socially regarded only as a work for young boys and not as a profession for adults) they tended to participate in it for short periods of time in their lives. So, within the practice there were always some newcomers arriving and some *ardinas* at different stages of competence or even changing roles. In this way we can say that their community of practice was not a rigid or closed one although a certain control of the access to it was carried out both by the two newspapers institution and the *ardinas'* community.

Another aspect that seems to be visible when we look carefully to the *ardinas'* practice is its **non-self-contained** character. That is, what was happening there, how participants behaved within it, how they read what was important or not, or what they used as resources for taking decisions was shaped not only by what was particular to that practice but also by the connections to adjacent practices. For instance, the presence of values was clear, sometimes conflicting but co-existent, coming both from the economical character of the practice and from the more traditional humanistic aspect of the culture of Cabo Verde that shapes the relationships between adults and children. Both those different values had a role in the constitution of the practice, shaping the participation and the *ardinas'* reading of what was important or not within it. As a social practice (that took place in a mainly socio-interactive and open environment) the social references were more visible than the mathematical ones. Therefore when *ardinas* needed there to decide about what was more appropriate, they relied more spontaneously on the former than the latter ones.

Another interesting aspect of this social practice that seems to play a role on the *ardinas'* learning is the **history** of the practice; how it passes to them, how they are aware of and appropriate it; how they re-write it and, in a sense, how it organises also their learning curriculum.

From these three aspects of the practice we can conclude that when the *ardinas* act in order to sustain participation in the social practice they are living moments of transitions (from newcomers to old-timers, from one role to the other, between different rules and values) within a certain historical recursive (but not equal) reproduction. These seem to us to be central issues that helped them growing as competent learners and not only as competent *ardinas*.

3.1.2. Ardin as participants (within a social practice)

An important aspect that emerged from the analysis reported was the pivotal role played by the *ardinas* effort **to sustain both their participation and the practice**. Even the main (and initial) goal of their participation being the need to help the family by earning some money some other particular motives (to feel the movement of the big city, to have money for gambling) shaped their approach to the practice. On one side, they wanted to maintain their acceptance by both the agencies but also by the community, so they needed to learn how to go on with both of them. This was the case, for instance, in the input they had (in their action as a community of practice) in the adjustment of the rules, but also how they individually adjusted their own behaviour to the possibilities they “read” as acceptable in each of the agencies. On the other hand, in their movement from newcomer to old-timer, some of them changed their roles, for example “teaching” one newcomer or being in control of a group of *ardinas*, or even being involved in activities within the agency. In this transition, some of them acted more strongly as the gatekeepers of the *ardinas* community and others more as “representative” of the agency. However, none of them put totally in risk their participation both individually and as a group, so in a sense they acted also in a way that ensured (to the agencies and the customers) the utility of the selling process of newspapers being done by a group of *ardinas*. In a way there was among the *ardinas* a sense of having some power both within their community of practice and in relation to the social role played by the newspaper in the society.

As we saw the *ardinas* as individuals have different motives for participating in the practice, but they also show different **ways of dealing with the resources** available within the practice. This is the case not only in the ways they appropriate the resources (as support elements for their calculations) but also how they connect them with other resources available (for example, their knowledge from school mathematics or from other newspapers selling activity). The analysis showed evidence that it was only when the *ardinas* get a certain competence in the whole practice — according to Lave and Wenger (1991) when they achieve full participation in the practice — that they are able to use the resources in a more effective way for “reading” the situations. This was the case, for instance, in comparing the two systems of access to the practice with more objective references that bring into play the fact that the income can be different in the two systems.

Additionally, the competence of being an *ardina* (in what it involves to be familiar with the usual calculations involved but also to know and accept the values of the practice as their own) makes a difference in **what they elect as a problem and how they respond to it**. For instance, when we were analysing their internal conversation it was possible to recognise the existence (for the *ardinas*) of different problems in situations they usually referred as the same “to check the money and newspapers”. So the meaning (in use) of this expression varied according to their competence. The strategies they used to deal with it were not always the same, it could be counting (money as newspapers) or calculating (the money they should have for paying back to the agency). It was the case for the newcomers, who carried few newspapers with them, that they did not buy anything before paying back the newspapers but they also knew that sometimes they lost money or newspapers. So, their need for checking usually came to their mind as a suspicion that someone took some newspapers from them or they could have given some change incorrectly to a customer. This is a newcomer problem and they use more a strategy of counting the total amount of money and newspapers they carry all together. On the other hand the old-timers sometimes do buy something before paying back to the agency and they are confident that they did not lose money or newspapers, so what they try to figure out when they are checking the money and the newspapers is to see if they have enough money to pay back the newspapers already sold. In this case they do not count or think of the total amount of money involved in the selling but only of the money they should have in order to pay back the newspapers. The focus of their problems is on being able to follow the rules of being a good *ardina* (paying back correctly). In this situation the calculations they do are much more complex, and show a shared repertoire of routines using what we called the same support elements for calculations.

The basis for knowledge (or for the re-contextualization) of (mathematical) facts seems to draw from the competence of being a participant (*ardina*). That is, being able to act in order to sustain the individual as well as the group participation and at the same time to sustain the practice itself, to elect what are their problems within the situations and according to their ability to use the resources from that particular practice as well as from other practices where they participate, to deal with those problems in an appropriate way (in what this means within the practice and for themselves). This seems to be a relevant issue if one wants to understand the ways students learn within their participation in school mathematics practice in the classroom. Learning to understand school mathematics practice and the role of that

practice in the students' life projects from the point of view of the learners is the starting point for the analysis of how students learn school mathematics.

3.2. On the research process

The methodological difficulties of gaining access to children's meanings are visible in the study reported. The fact that the research is studying a phenomena which was almost totally strange to us in most of its aspects, led us to realise that we had to go through a process which should involve, to a certain extent, our participation in the (*ardinas*) practice with the explicit (for us and for them) goal of learning it but not in order to be a full member of that community of practice. This starting point (more in terms of knowing that there are more things that we don't know than that we know) opened that community of practice to us but also gave us consciousness that methodological issues were central in this research.

First, there was the recognition that we were outsiders in a practice which was not familiar and that we couldn't understand it in full. There was several issues contributing to this: (i) the difference in languaging among the *ardinas* and when they talked to us; (ii) the fact that we were white people in a black community; (iii) the fact that there was difficulty at the beginning of thinking in terms of a radically different practice (selling newspapers against teaching mathematics), particularly with regard to the role of discourse within both practices. An important fact was that the researcher collecting data in the field was an adult woman and she was entering into a practice developed mostly by young boys and very few men.

Because there was a strange person in the group the need to create an emphatic relation with the group became relevant and explicit. But at the same time there was outside interference in the practice that was taking place. Because one really wants to enter in that practice, empathy is being created but this leads to a change in the practice – or at least in the way the practice is practised when the observer is present.

There is evidence that the young boys (observed within their practice as *ardinas*) work in different frames according to their motives, needs and values at the moment. By interacting with the observer as informants they were often brought into a frame different the one they used in their ordinary *ardinas* practice. In particular, when they tried to explain to the observer their processes in dealing with the calculations within the practice they tend to put themselves in a frame near that of schooling. This could have happened because it was the frame they imagined she could better understand (she was a woman as usually their teachers were, speaking more fluently in Portuguese, which is the school language) or because their practice was very little argumentative about those issues both within the community and with the authority (the people from the agencies who control their activity). In this situation they were acting and talking as informants in order to help the observer (an outsider) to make sense of what could be understood as the mathematical knowledge developed within that practice. Doing so some *ardinas* showed that they were dealing with two different definitions of the same problem but others acted as if there were two different problems. In both cases they were living a non-ordinary experience within their usual practice — talking about mathematical facts. They showed difficulties dealing with the need for re-framing their practice in terms of the frame of schooling.

This was clear in the discourse of the *ardinas* and shows the difficulties that one can expect when trying to analyse the ways children do school mathematics and to interpret their utterances. In these situations it can be more useful to look at those difficulties not as the child's inability towards the mathematics itself but emerging from the transition situation experienced by a particular child or a particular group.

At the same time, collecting data in the street is something completely different from doing so within the classroom. In fact, in the street both the physical and the social organisation of the practice (as that from the *ardinas*) is lived in a public and non-structured (for that practice) space with different frames constantly interfering at the same time. This aspect can present problems particularly for a teacher mainly used to the school context, but also constitutes an interesting opportunity to understand the transitions people live within their practices.

A final important issue relates to former – although recent – forms of colonisation of the country of the researchers over the country of the participants; this is something that people of both countries seem to be trying to solve through relations of work and cooperation. However it is an issue that would deserve scrutiny if one wants to see the implications of this kind of research.

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Notes

- ⁱ Culture, Mathematics and Cognition — Reflecting about Learning in Portugal and Cabo Verde. This project was supported by Fundação Ciência e Tecnologia under contract PRAXIS/PCSH/C/CED/146/96.
- ⁱⁱ *Ardina* is the Portuguese name given to those people who sell newspapers in the street. This was the way newspapers were sold for example in Lisbon until the eighties.
- ⁱⁱⁱ Portuguese is the official language in Cabo Verde. Therefore, it is for example used in the school and it is the language of the newspapers. However, Creole is the spoken language used in everyday activities. Far less people are really fluent in Portuguese.
- ^{iv} Several factors contributed to the fact that these boys abandoned the practice of selling newspapers in the street: some of them had the opportunity of working in S. Martinho helping the construction of infra-structures in the village organized by the local authorities.
- ^v This adult had been one of the *ardinas* when he was a boy and for some years he was involved in the work with the *ardinas* but in no way that had not much visibility in institutional terms. He had not a working place in the agency of the newspaper and he only worked for the newspaper in the day when it was being sold. Besides that he lived in S. Martinho and had friendship links and even familiar links to some of the *ardinas*.
- ^{vi} The *ardinas* use the expression “to be a good ardina” to classify those who “follow the rules” or who are honest, serious, relating it mostly with the payment process. We will show later on (point 2.2.2.) how this value is present in their practice and how it interferes with their interpretation of the profit.
- ^{vii} This way of organizing the records was not learned from Disidori. In fact Manu told us that he reminded this process from a boy who did the work before Disidori.
- ^{viii} Buying means that the *ardina* should have enough money to buy the newspapers at the moment he receives them; all newspapers left after the selling are not returned to the agency and consequently not refunded. Taking refers to the process already in practice in 1998.
- ^{ix} Piratas (pirates) is the word in Praia to refer to kids who make small robbery in the streets.
- ^x Legitimate peripheral participation is proposed by Lave and Wenger as a “descriptor of engagement in social practice that entails learning as an integral constituent” (1991, pg. 35).
- ^{xi} The dialogues between the observer and the *ardinas* were in most cases a mixture of Portuguese and Creole. Generally the *ardinas* expressed themselves in Portuguese with difficulties and often they jumped very fast to Creole. The translation from Creole or Portuguese into English presented in this report certainly transforms the original meanings as in every translation process. We tried to keep the translation of the *ardinas*’ utterances as close to the meanings as possible with sacrifice of the elegance of the wording in English.
- ^{xii} In the case of the younger *ardinas* (like Djeps at the start) there is also the 5 or 15 which are quantities handed over to the younger ones when they begin to participate and are learning from an old-timer.
- ^{xiii} *Conto* is a Portuguese that stands for one thousand escudos. It is used by some of the *ardinas* to refer to one thousand escudos from Cabo Verde.
- ^{xiv} Although those words are commonly used by most people, they are strongly related to the *ardinas*’ practice for this is what they use when they describe what Disidori or Manu do with the calculator in order to say them how much they should pay - for example, “50 [newspapers] times 87.5 escudos”.
- ^{xv} This idea of “blindly consciousness” is how Bloor, 1997, pg. 51, summarized how he read Wittgenstein expression of “obey the rule blindly”, Wittgenstein, 1992, §§: 219
- ^{xvi} “A learning curriculum consists of situated opportunities [...] for the improvisational development of new practice. A learning curriculum is a field of learning resources in everyday practice viewed from the perspective of learners.” (pg. 97).
“Production activity-segments must be learned in different sequences than those in which a production process commonly unfolds, if peripheral, less intense, less complex, less vital tasks are learned before more central aspects of practice.” (pg. 96).
- ^{xvii} Understanding power as “collective agreement conferring authority to” (Wenger, 1998, pg. 15).
- ^{xviii} For instance, when O Espaço newspapers are not ready in time *ardinas* like to help the workers inside the board even without payment for that work, but they do not do the same with O Tempo (in 1999). Similarly the *ardinas* behavior towards Espaço and Tempo related to the payment are different. Old-timers sooner or later stop being serious to Tempo regarding the payment of newspapers sold but never to Espaço.

^{xix} Santos and Matos (1998) discussed and analyzed the difference between structuring resource and cultural artifact and their role in school mathematics learning.

Key words: community of practice, competence, internal conversation, mathematics, newcomer, participation, old-timer, situated learning, social practice, transition.