Semiotic Mediation and Metaphor in Teaching and Learning Mathematics

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Overview
Investigations into the teaching and learning of mathematics require theorisations of these processes. How do children learn? What is the role of the teacher? What are the particular problems posed by setting these questions within the context of mathematics? What methodologies are appropriate to a theory? The literature of mathematics education research reveals a range of theories being used and developed (Tsatsaroni, Lerman & Xu, 2003). This proliferation calls for researchers to address the issue of what resources any particular theory offers that are relevant to the task. Our purpose in this chapter is to identify these resources in relation to the sociocultural theories of semiotic mediation and metaphor and demonstrate how they can inform our understanding of mathematical thinking and learning.

We start from the fundamental premise that consciousness is mediated. Meanings are appropriated from others in a dialectic of prior experiences and identities and new experiences and identities, within social practices. Mediation begins from the very earliest stages of the child's life. Minick (1987) draws on Vygotsky to look at this process:

He argued that when the infant cries or reaches for an object, the adult attributes meaning to that behaviour. Though the infant has no communicative intent, these acts nonetheless function to communicate the infant's needs to his caretaker. Here, as in the adult's attempts to interact with the infant, the infant is included in communicative social activity before he has the capacity to use or respond adequately to communicative devices. Vygotsky argued that this provides the foundation for the transformation of the infant’s behaviours into intentional indicative gestures. (p. 28)

Wittgenstein (1967) describes the profound effects of the process of mediation as follows:

Words are connected with the primitive, the natural, expressions of the sensation and used in their place. A child has hurt himself and cries, and then adults talk to him and teach him exclamations and later sentences. They teach the children new pain behaviour. "So you are saying that the word 'pain' really means crying?" -- on the contrary: the verbal expression of pain replaces crying and does not describe it. (Wittgenstein, 1967, p. 89)
Thus meaning-making becomes a central focus in studying consciousness and in studying the process of learning in particular. We begin this chapter by looking at metaphor as a special case of meaning production, one that has great significance for mathematical thinking and learning. Following a first approach to the role of metaphor we look at semiosis as a tool for examining the evolving process of meaning production as socially and culturally mediated. Finally we will return to metaphor and look at it through these theories, combined with Paul Ricoeur’s perspective on the iconicity of metaphor.

**Metaphor as a special case of meaning production**

The contemporary theory of metaphor (Lakoff, 1987, 1993, 1995; Lakoff and Johnson, 1980) moves away from the rhetorical tradition, which emphasises the stylistic aspects of metaphor. In rejecting a view on metaphor as an ornament of speech, Lakoff and Johnson (1980) claim that our conceptual systems and consequently all human thinking are metaphorical by nature.

A conceptual metaphor is defined as a correspondence between two conceptual domains. It consists of a mechanism that allows us to understand a conceptual domain in terms of another, usually more familiar or closer to our daily experiences. In the words of Lakoff (1993) this correspondence is a real mapping or projection from an origin-domain onto a target-domain. In the extensive research carried out by Lakoff and Johnson (1980), metaphor reveals itself as a conceptual phenomenon that is largely responsible for the way we conceptualise the world. As a fundamental attribute of human thinking, metaphor can be characterised by a set of properties:

- Metaphor is essentially conceptual and not merely a figure of speech;
- Most phenomena from the more trivial ones to those which are addressed by complex scientific theories can only be understood by means of metaphors;
- Metaphor is the major mechanism through which we can make sense of abstract concepts;
- Conceptual metaphors form a system in permanent use, even though that system usually operates in an unconscious, automatic and effortless way;
- The first basis that supports the correspondences we naturally produce between conceptual systems is our biological relationship with the world; furthermore we use our daily experiences in the production of metaphors.

The work on metaphor by Max Black (1962, 1993) addresses how the interaction between the two topics or domains might be performed. Each of the two topics works as a system rather then just a
number of disconnected elements. The presence of the primary topic in a metaphorical statement induces the selection of particular properties and attributes of the secondary topic, which will shape and generate a complex of implications. This set of implications could be regarded as a kind of common knowledge about the secondary topic. Then the metaphor operates a projection of those implications onto the primary topic. As an outcome, a parallel complex of implications appears within the primary topic. These implications, however, must be coherent and adjusted to the topic where they become integrated. The fundamental result of the metaphor is the selecting, emphasising, suppressing, and organising of characteristics of the primary topic, suggesting and stressing ideas about it that would normally be applicable to the secondary topic.

For example, in the metaphor MORE IS UP AND LESS IS DOWN, we obviously are aware that more is not really up. Still there are some features of up that can be applied to more and features of down that can be applied to less, such as "the prices are going up" or "turn the radio down".

As with many conventional metaphors, this example shows that a conceptual metaphor inherits a direct and experiential basis that ends up working as an indirect experiential basis in our understanding of the world. In other words our own experience with the objects that configure our daily experiences provide the sources for grand metaphors capable of turning into actual paradigms of thinking. These grand metaphors, such as MORE IS UP AND LESS IS DOWN, are instantiated in many everyday metaphors such as those above.

An important aspect arising from this perspective is the notion that a metaphor works as a kind of filter. It mediates a correspondence in such a way that the secondary complex of implications can be seen as a model of the properties and characteristics of the primary topic. Thus metaphor is granted a powerful cognitive function. Whenever we use a metaphor, two thoughts on separate topics are kept in mind and act together, under a certain word or expression.

Although the rooting of metaphors in a bodily experience has been asserted (Lakoff & Johnson, 1980; Lakoff and Nuñez, 1997) their permeability to culture and social aspects is perhaps as relevant and powerful.

Our physical and cultural experience provides many possible bases for spatialization metaphors. Which ones are chosen and which ones are major, may vary from culture to culture.
It is hard to distinguish the physical from the cultural basis of a metaphor, since the choice of one physical basis from among many possible ones has to do with cultural coherence. (Lakoff & Johnson, 1980, p. 19)

Thus we might note 'bigger is better' and 'small is beautiful', or 'argument is war' versus 'argument is a dance' (ibid, p. 5) as examples of how different cultures would be expressed in contrasting metaphors.

**On metaphors in mathematics education**

In spite of the current remarkable attention devoted to the study of the nature and functioning of metaphor, particularly in the field of cognition, there are prevailing questions and controversies about its importance and role in education.

One of the arguments in favour of the use of metaphors in education is the notion that they yield the transfer of learning and understanding from well-known experiences to the less familiar, in a vivid and retrievable way.

To understand a concept with resource to metaphor is something completely different to subordinating such a concept to a fixed and single meaning, either through a description or a definition. The metaphor demands the creation of connections, it suggests meanings, indicates resemblances, unfolds correspondences, highlights certain features and ignores others. Thus metaphor can be a platform upon which the understanding grows; it coincides neither with the concept nor with some possible definition, nor even with the meaning of the concept. The metaphor is not a way to grasp the meaning; it is rather a way of producing meanings.

The view expressed by Nolder (1991) is that metaphors permeate a whole range of dimensions within the teaching and learning of mathematics. In spite of that, it is possible that many teachers are not aware of metaphors in their teaching or think of them as irrelevant based on a belief that, in mathematics, things are exactly what we say they are. In demystifying such illusion, Nolder notes that there is a human inclination towards creating correspondences and parallelisms, which are processes found at the very heart of metaphor. This is the reason that can explain the variety of expressions produced by students who struggle to express their ideas and to communicate what they have in mind or to develop new implications from a certain concept into others. One of the interesting examples presented is the following:
Michelle suddenly noticed that complex numbers were "like surds", checked on how to divide by surds which she only dimly recalled, applied the same method to complex numbers and found that it worked. (Nolder, 1991, p. 106)

But the role of metaphor in learning can also be seen in other ways as Sierpinska (1994) remarks. A metaphor may have two different functions: in one of them, metaphor serves understanding, in the other, metaphor serves explanation. In the first case, we can think of the spontaneous use of metaphor in face of a problem situation where something new is found and there is no direct way of describing it. Then, the metaphor will give an anchorage point onto something known to understand something new. When looked from another perspective, metaphor can be more a way of explaining. In this case, its role is to make accessible one or more details of a certain body of knowledge.

The ideas emerging from the work of Lakoff & Nuñez (1997) contain several implications that are of clear relevance in the plane of mathematics education. Perhaps one of the strongest is their claim that regardless of all the efforts to give mathematics a rigorous and formalist dimension, independent of the nature of human reasoning, mathematics and mathematicians can not do without metaphorical ideas like the number line or the natural continuum sense for a continuous function.

Since a great many mathematical ideas are metaphorical, teaching mathematics necessarily requires the metaphorical structure of mathematics. This should have the beneficial effect of dispelling the myth that mathematics is literal, is inherent in the structure of the universe, and exists independent of human minds. (Lakoff & Nuñez, 1997, p. 85)

In the field of mathematics education, mathematical models and metaphors have also been the subject of some inquiry. The apparent closeness between model and metaphor stimulates the approach to mathematical models and modelling from the point of view of metaphor. Krumholtz (1989) has referred to such connection by considering models as metaphors. The basis for such a position is the fact that in producing a mathematical model we would be creating a web that connects and brings closer different conceptual domains. Through such a web transference between separated domains becomes possible. So in the mathematical modelling of real phenomena a space of meanings of mathematical nature is produced that allows one to read and to reinterpret the real situation.
Matos & Carreira (1995, 1996) have extended this view to underline the interactive nature of metaphors when associated with the production of mathematical models. They have argued that mathematical concepts and ideas involved in a model have a dynamic and changeable nature. Based on classroom episodes of creating and exploring mathematical models, they have described a mathematical model as a dynamic system of meaning transference. The conceptual structure of a mathematical model would then be dialogical, in the sense of a dialogue between conceptual systems.

A mathematical model reveals itself as a model of something when its metaphorical matrix is unpacked and put into action (Carreira, 1998; Carreira, 2001). Metaphorisation becomes, as such, an essential part of thinking mathematically and of the process of making mathematical ideas mean something both within mathematics and outside it. As students work on mathematical models of different phenomena the metaphorical matrix of a model works as a mathematical thinking tool. Mathematics can therefore work as a way of 'seeing as'. The meaning of mathematical concepts and processes becomes a double-anchored meaning. Problem situations that involve the use of mathematical models present opportunities to put together in interaction different conceptual domains. Applied problems, in entailing the use, application or construction of mathematical models, drive and stimulate the production of metaphorical meanings. We will return to this in Chapter ** where the analysis of a sample of the data collected in the study will take us to the discussion and re-examination of the notion of transfer in mathematics learning.

When we speak of unpacking a metaphorical matrix above we are directing attention to who does the unpacking and what constitutes appropriate unpacking in any specific context, and hence the notion of mediation. For the moment we will just suggest that at the core of metaphorisation is the invitation to interpretation and to the production of alternative views. In that sense, it means an extension of interpretative chains, which we address in the next section.

**Mediation: The denial of neutrality in meanings embedded in social practices**

As we begin our discussion of mediation we find ourselves searching for an appropriate illustration which might capture the sense we wish to convey. As a first attempt we can take the idea of conveying food from one place to another with, say a spoon. A first reflection might see the role of the spoon as neutral, the only important features being the food, the original place, such as a plate, and the final place, perhaps the mouth. The spoon appears to play no role other than that of conveyance. In a similar manner the teacher, or parent, might be thought to convey ideas to the
child, the key elements being the knowledge and the mind of the child. A second reflection, though, could be stimulated by considering an alternative tool for conveying food to the mouth, such as the hand, as in eating Indian food. It then becomes clear that the spoon, or hand, do play a role in the act of experiencing the food. The mediating object becomes part of the meaning of food and the social practice of eating although we might become aware of the significance of the mediating object only when we consider an alternative; that is to say, the mediating tools become tacit in practice and only become explicit when attention is drawn to them. Thus we become aware of the spoon's rigidity, its feeling in the mouth, how one holds the spoon, the lack of sense of touch of the food itself, and so on. This is the sense of mediation we use and our example, drawn from the culturally and socially situated customs and meanings associated with food and eating, is a particularly rich context to serve as illustration. Metaphors abound in relation to food, such as "We don't need to spoon-feed our students" and "There are too many facts for me to digest them all" (Lakoff & Johnson, 1980, pp. 46/47).

As with the 'neutral spoon' idea, we might see scaffolding as neutral. It is usually interpreted as the teacher designing tasks and materials to provide the assistance necessary for the child to move from her current understanding to a stage the teacher wishes, and the job becomes one of taking the child through those stages. Like the spoon, the teacher plays no role other than conveyance. Traditional teaching styles appear neutral in terms of the role of the teacher in transmitting knowledge to pupils. Arising from the second reflection on mediating food we might say, in relation to learning and teaching, that the teacher, the text, the activities, the didactic contract, other students, and so on are integrally involved in the knowledge that is acquired.

From that perspective, we need to look at school knowledge and to the way teaching and learning become integral parts of a more general process of cultural and symbolic mediation. Our attention is therefore directed to the particular aspects of the pedagogical scenario that become tools to act upon and to transform reality. Just to stress this particular point, we may take a common example of a mathematics class. A mathematics teacher brings to the classroom several objects. There is a football, a billiard ball, an orange, a balloon, a CD, a ball of wool, a light bulb, and an earth globe. "Which of these objects would you say is a sphere?" asks the teacher, having passed the objects around the group of students.
A number of questions can arise from the particular pedagogical situation including, amongst those of most relevance to this chapter, questions on the nature of the relationship between the subject and the object of knowledge in a school mathematics setting. Why does the teacher bring to the class the referred real objects? What makes them susceptible to being differentiated and grouped? Under what conditions is it reasonable to sort them by groups? What are the differences between each of the objects presented and the ideal, formal geometrical concepts of sphere and spherical surface? What possible answers could be expected? Is the billiard ball the closest image of a sphere? Why? Could we look at the orange peel as an image of a spherical surface? What are the signs that mediate the interactions undertaken in such a plausible mathematics school setting?

From a Vygotskian point of view, the process of learning the concept of sphere, as an instance of a scientific concept, is a process of mediation between the subject and the object of knowledge. The interaction between the individual and the real world is regulated and transformed by the use of symbolic material. The ways of organising the world that are created by culture, within a social and historical context, become internalised by the person under the form of symbolic matter. Our access to the real world is not exclusively derived from direct experience with reality. Such access is provided through the forms of reality that are given by the symbolic systems available within the particular cultural context of learning, and hence is inevitably socially organised. It is the culture that offers individuals, differentially according to their social group identity and background, the symbolic systems of representing reality, and through them, the universe of meanings that allow one to interpret and to organise the data collected from the real experience in the world.

The relationship between the development of higher psychological functions – such as concept formation – and formal school learning has been studied by many of Vygotsky's followers. Cole (1990) describes cultural mediation as the living in a double world, both "natural" and "artificial". Human beings live in an environment that although containing natural elements is continuously transformed by means of artefacts that are produced by successive generations through time.

Mediation, then, is not neutral, and different pedagogic modes constitute different forms of mediation. Teaching, whether the intentional activity of the teacher or the unintentional activity of a parent, other adult, or peer, is therefore to be seen as central in all learning: one can only learn from others. We do not, however, want to argue for a reductionist position where individual behaviour is

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1 Note that in this volume we use transmitters and acquirers to stand for teachers/parents/more able peers on the one hand and students on the other. We are aware of the pejorative implications of these terms in the
only the internalisation of the socio-cultural. Instead we will outline in the next section a different interpretation of individuality which locates it in each person's unique network of prior (sociocultural) experiences.

**Semiosis: Potential progression and infinite regression**

Through a discussion of semiotics we will articulate the notions of mediation and meaning-making with the individual's interpretation of any experience.

Semiotics is the study of signs and sign functions in all aspects of message exchange and it concerns the conveyance and development of meaning through all sign vehicles. (Vile, 1996, p. 26)

We will refer here to Peircian semiotics rather than Saussurean (see e.g. Walkerdine, 1988 for work drawing on Saussure). A fundamental assumption in Peirce's semiotic work is the fact that meaning is a result of a triadic relationship where the act of interpretation plays a decisive role. Every sign consists of a structure involving three elements in interaction: the representamen, the object and the interpretant.

![The three elements of the Peircean sign](image)

The first element is some physical stimulus that can be detected through the senses, such as a written or spoken word, a traffic light, a person's gesture, a footprint, a smell of smoke, the taste of a wine, a ring of a bell, etc. The second element, the object, although not a physical object, is what the representamen stands for or what it is taken for. To give an example, suppose the ring of the school bell stands for the time to end a lesson. The third element is the interpretant, that is, what makes the sign mean something to a particular individual, in a particular context. For a student in a class, the ring of the school bell may mean the end of the class, but it may also mean the need to
pack his or her materials before getting out, or it may mean that he or she will meet a schoolmate in a few moments, or that his or her work on the given tasks was not completed in time. For the teacher it may mean the need to rush the conclusion of the class, it may also mean the time to give students final instructions regarding the work ahead, or it may mean the possibility of going to the cafeteria and having a sandwich before the next class. A similar bell might be a fire alarm. This bell would not be at an appropriate time, may not sound exactly the same, and so on. This second story indicates that representamens are always social/cultural products, and that the way a representamen could be interpreted would have something to do with the social expectations/experience of an individual.

The triangular structure of semiosis is suitable as a schematic frame since it moves away from any assumption on a pre-given ready meaning and rather highlights that a sign or representamen is something that stands to somebody for some object in some respect or capacity. In order to become a sign, any stimulus must be involved in an act of signification, the act in which it receives an interpretation and determines another sign – the interpretant of the first sign – to the same object.

A sign can only become clear in its meaning when it sends to an interpretant, which in turn sends to another interpretant and so forth ad infinitum, in an unlimited semiotic process in which the receptor decodes the original sign (Eco, 1981). In Vile’s words “a sign becomes significant only through its relationship with other signs and thus the meaning of a sign is virtual, only retaining that meaning by virtue of the possibility of being inserted into further acts of semiosis as a sign” (1996, p. 43-44). The unlimited semiosis, that is the unfolding of interpretants, is the process by which a sign may refer back to another sign or string of signs and at the same time can become a reference for a future sign.

The concept of infinite semiosis seems to be adjustable to some philosophical perspectives on the nature and the development of mathematics. The work of Lakatos on the quasi-empiricist nature of mathematics is one of such views in that mathematical objects are created in specific social interactions (such as the original definition of a polyhedron (Lakatos, 1976)) and become objects on which others can act. This may lead to one of a variety of developments that change the definition or might even lead to its use from a different point of view, or to its being reconceptualised. Meanings are never closed; there is always potentially an infinite semiosis. Others, within the field of mathematics education, have pointed out some aspects of mathematics that are in harmony with the semiotic infinite regression, for example Freudenthal (1983).
To speak of a chain is a form of emphasising connection and continuity. As an example, we could take the meaning of even number. There is the interpretant consisting of any integer divisible by two. There is also the idea of generating the even numbers as the multiples of 2 (2n, n integer); or else a number that can be divided into two equal integers; any integer whose last digit is 0, 2, 4, 6, 8. Moreover, in a particular semiotic system of signs as in the case of the Portuguese language, the word "par" is used both to signify even and pair. The word "par" would be used as in «the number 10 is "par"» and in «here is a "par" of shoes». To say that a number is "par" (even) can be interpreted as it representing a certain number of pairs. 10 shoes is an even number of shoes because it is 5 pairs of shoes. The sum of two even numbers must be an even number since it is the sum of a certain number of pairs with another number of pairs, resulting in a new number of pairs (2n+2k= 2(n+k)). The product of any number by an even number is an even number, meaning that the multiplication of a certain number of pairs by some factor yields a new number of pairs (m.2n=2(m.n)).

We could continue listing interpretants for even numbers but to conclude the example briefly sketched we would like to note that there is no such thing as the meaning of even number. What would we state as the meaning of even number? The several propositions made on evenness or a more general one that could suit us as a kind of definition? Is it possible to have meaning subsumed under a definition? In asserting that meaning making is a process, rather than the acquisition of a ready-made conceptual structure, the meaning of evenness can only be seen as having an evolving nature and the locus of meaning can only be placed on the cultural mediation between subject and object. Perception of the object is always mediated by the culturally grounded meanings produced by the chaining of interpretants.

From the point of view of the teacher, some interpretants are seen to be more useful and appropriate for legitimate mathematical thinking than others. In a study of year 8 students writing about algebra, Vile (1996, p. 128/9) gives an example of a student who, on seeing the equation <x - 5 = 3> explains in a letter to a fictional friend that the symbol <x> is in place of the number <8> because <8 - 3 = 5>. Vile concludes that the student is using generalised arithmetic. In this case, <x - 5 = 3> is the object, <x> is the sign and <unknown number> is the interpretant. The student might have written that whenever you see a –5 you must add 5. The interpretant here could be <do the inverse of this> and we might want to say that this interpretant might carry other meanings or the potential for other actions that are unsuitable – even wrong! This immediately raises questions such as: how
are interpretants acquired/generated; by whose criteria are 'more appropriate' interpretants determined; what kinds of intervention can a more informed person make? The first question is one we will address here. Other questions are addressed in other chapters in this book. From our perspective Vygotsky's notion of semiotic mediation, developed further by Wertsch and others, provides the key element of development, the acquisition and generation of interpretants in a cultural context, not addressed by Peirce.

Semiotic mediation

The notion of semiotic mediation brings together the social origins of consciousness, the function of language as cultural tools and the meaning-making process based on prior network of experiences captured in the semiotic triad of Peirce.

As in the learning of a second language, the process of learning scientific concepts in a formal instructional setting stands on a range of meanings previously developed and originated within daily experiences. Such knowledge spontaneously produced becomes a mediator in the learning of new knowledge.

The development of a spontaneous concept must have reached a certain level for the child to be able to absorb a related scientific concept. (Vygotsky, 1986, p. 194)

A fundamental issue to be taken from Vygotsky's words is that scientific concepts require for their production and development a certain basis of spontaneous concepts. Furthermore, one important condition for the teaching and learning of scientific concepts is the need to make them consciously the object of thinking, manipulation and use. The learning of a scientific concept is not a straightforward process. It takes a history that eventually will place it within a conceptual system. Its function is not the 'defeat' of spontaneous concepts, as poor and distorted structures to be overcome, but instead it will rehabilitate such concepts as equally powerful in the active and often creative production of meaning. For every scientific concept, the relationship with objects is mediated from the very start by some other concept. The notion of scientific concept entails the idea of a position in relation to other concepts, that is, a place within a conceptual system.

Concepts are evolving structures that are not endlessly kept in some mental reservoir. The change of a concept is produced as it becomes connected to other concepts and, ultimately, it means the evolution of a conceptual system. As the system develops, each concept takes a different position and, as a result, it represents a new understanding. Vygotsky offers examples to illustrate how a
concept begins to be part of a system and argues that such is the moment when it can be an object of thought and deliberate control. A child learns the word "flower" and soon after the word "rose". For a certain period of time, the concept of flower, being more general than the concept of rose, is not perceived by the child as having a different degree of generality. Both words are used indifferently and their meanings are juxtaposed. When eventually the idea of flower becomes more general, it means that the relationship between flower and rose and also with other linked concepts has changed. A conceptual system is being configured.

In the illustration of the sign functioning, we may now consider an example given by Eco (1976/97) where a scientific definition of lemon is distinguished from a common definition of lemon. In a scientific definition of lemon there may be elements that send to semantic units like vegetable, fruit, citrus, citric acid, corrosive, and so on. What characterises this type of definition is the hierarchical structure of the connections undertaken. If there would be a limit point and we could do the inverse, without previously knowing the meaning of lemon, we would be able to find out what the sign "lemon" stands for. Those are interpretants belonging to cultural units supposedly universal in terms of a code that belongs to science, Biology, Botany, Chemistry, and so on.

A common definition assumes several possible interpretants in accordance to the context and belonging to several different cultural units, through a series of connections. The lemon sign can perhaps be sending to fruit, yellow, juicy, refreshing, quenching one's thirst, healthy, Mediterranean climates and so on. Each of the components of the series could be related to other cultural units and therefore other connections would produce a diverse but culturally grounded meaning for lemon.

In the learning of concepts the use of signs constitutes one of the most important ways by which the cultural forms come between the individual and the world, and in becoming the 'filter' through which the individual is able to understand the world and to act upon it. (Oliveira, 1992). That is not to say that culture is a static and rigid system to which the person submits blindly. On the contrary, the elements of a culture are in a permanent process of recreation and reinterpretation of data, concepts and meanings. Thus, we might add to the list above the discovery that lemons break down grease when squeezed on oily fish, which gives a meaning to why one receives a piece of lemon with a fish dish.

Signification is the creation and the use of signs, that is, a process by means of which human beings remove themselves from the real world. The idea that language has withdrawn us from immediate
reality is quite central in Vygotsky's semiotics, since it justifies the exclusive human behaviour derived from the creation of artificial signalising stimuli: the process of signification.

Indeed Vygotsky sees meaning as the meeting point between thought and speech and describes it as the synthesis of two basic functions of language: the social interchange and the generalising thought. At first, the word is primarily a simple indicator (an index). It is where all semiotic mediation starts via the social interchange. The purpose is to draw attention to a particular object. Only later, the indexical character of signs gives way to its symbolic character, the generalising thought. The condition of spatial-temporal contiguity between the sign and the object is progressively released and the distance between referent and sign increases. Other mediating means fill the gap in the meantime, through the creation of links that relate words to objects, to other words and to other objects, as illustrated by the phenomenon of infinite semiosis.

The acknowledgement of the transformation of meanings is stressed in Vygotsky's thinking through the assertion that the acquisition of a new word represents the beginning and not the end of its meaning. He also claims that the experience brought by formal instruction represents an important element in the development of genuine concepts. "Vygotsky argued that experience in educational activity is an important force that guides the development of genuine concepts, hence his distinction between the genuine or 'scientific' concepts learned as a result of schooling and the 'everyday' or 'spontaneous' concepts learned by the child elsewhere" (Wertsch, 1985, p. 102). The sign-object relations are underlined in the case of spontaneous concepts, suggesting its dependence on the context, and the sign-sign relations are distinctive of the scientific concepts, implying the release of the concrete reference and the decontextualisation of meanings.

When forms of generalisation are produced, the concept is both related to an object and to some other concept, by several possible internal and hierarchical links, which make the concept part of a wider conceptual system.

Coming back to the example of the teacher who uses an assortment of real objects to introduce the concept of sphere, and spherical surface, we may recognise some aspects of a conceptual system being produced. There are real objects that allow for the production of sign-object relations. Some of the objects are used as a fairly good reference for the word "sphere" and some not. The indicative function of speech is obviously present. There is a pointing to the objects that would be seen as spheres. But soon other concepts and meanings start to play their function. The CD looks more like a circle, although it is also round. Being round is not enough. The football is not a sphere, because it
is only approximately round. It has edges, which are not present on a sphere. The fact that it rolls is not enough to be a sphere. The ball of wool is not really solid, and the same is true for the electric bulb. Any section produced by a plane on a sphere would result in a circle. That's why the balloon is not a sphere. We could continue.

"With the development of scientific concepts, a child not only can use words such as 'table', 'chair', and 'furniture' appropriately in connection with the objects to which they refer, but the child also can operate on statements of logical equivalence, nonequivalence, entailment, and the like, such as 'All tables are furniture'. Hence the emphasis has shifted away from those aspects of linguistic organization that involve contextualization to the capacity of linguistic signs to enter into decontextualized relationships, that is, relationships which are constant across contexts of use". (Wertsch, 1985, p. 103).

In conclusion, we could say that conceptual development is the product of a gradual decontextualization of signs, or rather the result of the capacity that words have of getting in connection with other words in the absence of the actual objects to which they refer. The origins of conceptual development go back to the contextualized signs, which means, to the indicative function of language, mainly standing on sign-object relations. But the mechanisms of semiotic mediation move on towards the incorporation of increasingly more sign-sign relations that inform the symbolic function of language.

Before returning finally to metaphor, we want to point to two criticisms of Vygotsky’s work which are not solved by adopting a Peircean point of view: that there is no problematisation of culture; and that new tools, namely sociological tools, are needed to study the differential appropriation of scientific concepts. Regarding the former, on the one hand it is no surprise that Vygotsky did not engage with a theory of culture(s), given the time and place in which he worked, Russia just after the revolution. On the other hand Vygotsky’s personal history might have made him keenly aware of the role of culture in identity but we might surmise that the political climate of the time was not conducive to discussions of cultural difference. In particular there could not have been any consideration of the ways in which social practices might impact on different social groups. By drawing attention to these concerns we want to point out that other chapters in this volume can better engage with these issues, in particular theories of symbolic control (chapter **).
To this point we have outlined a notion of semiosis that combines with Vygotsky’s analysis of the function of signs and cultural tools in mediating consciousness. Peirce’s theory provides a fruitful structure and image of the internalisation process sketched by Vygotsky although we are not suggesting that Peirce’s theory is the only way of instantiating this process. With these tools we revisit the notion of metaphor and we incorporate into our view of metaphor the semiotic process whereby the duplication of interpretants functions to create meaning.

**Metaphorical meaning as semiotic process**

To think metaphorically is to think by means of duplicated referents. The analogy comes from the approximation of two interpretants, which in turn generate the nearness of two objects. The theory of duplication as a semiotic model of metaphor can be portrayed in the form of a *semiotic trapezoid*. Once we assume the role of two referents and two interpretants in metaphorical meaning, we can postulate an interactive relationship between these elements. Thus semiotic mediation in metaphorical thinking would be consistent with the trapezoidal model described in figure 2:

![Figure 2. Duplicated referents and interpretants in a semiotic trapezoid](image)

In facing the statement *TIME IS MONEY* one is confronted with a semantic conflict to the extent that everyone knows that time is literally not the same as money. To work out the semantic conflict entails the destruction of the literal sense of money that one applies to things like coins, bills, bank cheques, bank accounts, currency, or credit cards. Everyone is aware that time is not *really* that! To break up the conflict means to unveil the iconic character of the word money in the statement.

Paul Ricoeur (1983) highlights the iconicity inherent in metaphors by conceiving the *iconic momentum* of metaphor. He stresses the fact that the icon is not rendered by the metaphor. A metaphor is rather a formula for the creation of icons. Thus, metaphor is not synonymous with image since metaphor does not immediately display the resemblance. The metaphor works
iconically by indirectly unveiling a resemblance. And because it is not an image it has the power of pointing out new similarities.

In metaphor a semantic conflict is generated. But the important thing is that metaphor is not itself the conflict. Metaphor is the resolution of the conflict. Overcoming such conflict requires the setting up of a parallel between situations, a parallel that will guide the iconic transference from one situation to the other. Therefore the semantic collision is just one side of a process that has iconicity as the other side (Ricoeur, 1983).

In the example above, it is, after all, the possibility of creating a new sense for time, to see time as money in a certain sense, a sense that fits some of the properties that can be recognised in money: something that can be spent, lost, invested, saved, negotiated, etc. Time becomes similar to money by virtue of the metaphor. The ruin of the primary reference (bills, coins, cheques) gives way to a secondary reference and as a consequence two objects that were distant end up together.

![TIME IS MONEY](image)

Figure 3. The semiotic trapezoid in the metaphor "time is money"

The metaphorical statement triggers the analogy, but rather than being its cause or reason, analogy is the result of the metaphor. The use of a metaphorical predicate is comparable to a screen, a lens or a filter that selects, eliminates and organises the signification of a certain domain. Metaphor thus enables a semiotic mediation process in which we can envisage the chains of meanings being produced in learning, and also how the interaction between spontaneous and scientific concepts may be portrayed.

Some research issues
The approach outlined in this chapter lead us to pose a number of questions, by no means exhaustive, and we have grouped them by their main motivation.

Methodological issues:

- Can one find evidence of mediation taking place in teachers' actions, such as the overlaying of children's inaccurate mathematical language, or introducing particular cultural/psychological tools?
- Can one find evidence of internalisation, the outcome of mediation?
- Can one identify chains of semiosis in classroom talk? How does that reflect the development of concepts, in principle never completed?
- Can one recognise processes of decontextualising meanings, or sign-sign foregrounding?

Empirical studies:

- How does the teacher bring about preferred meanings? What happens when students’ meanings come from separate domains from the preferred meanings?
- What examples can one see of the interaction between scientific and spontaneous concepts in the mathematics classroom? What methods of dynamic assessment are available to ‘measure’ progress from spontaneous to scientific concepts?
- To what extent can a focus on metaphor enhance transfer?
- What risks are there in the introduction of metaphor in regard to emotion?
- Is there a role for teachers creating metaphors that they deem helpful to teaching and learning? How might teachers evaluate the ‘efficiency’ of students’ own metaphors?

Epistemological questions:

- What are the cultural sources of scientific concepts? How do these cultural sources impact on students’ metaphorical thinking?
- Are there scientific concepts in everyday concepts?
- To what extent are source and target domains separate in thinking prior to their connection in a metaphorical linkage?

References


