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PROMOTING MATHEMATICAL COMMUNICATION IN THE CLASSROOM: TWO PRESERVICE TEACHERS' CONCEPTIONS AND PRACTICES

ABSTRACT. Recent reforms in mathematics education have encouraged teachers to engage their students in various forms of communication. Scholars have begun to consider questions such as: In what ways do teachers facilitate and guide classroom discourse? How does the quality of students' reflections impact the development of rich mathematical understanding? In order to address these and similar questions, the authors provide a framework of four constructs that can be used to analyze various forms of classroom communication: *uni-directional* communication, *contributive* communication, *reflective* communication, and *instructive* communication. Throughout the article, the authors both develop and use these constructs as they consider two preservice teachers concepts of communication and their corresponding classroom practices.

KEY WORDS: mathematics teacher education, mathematical communication and discourse

Reforms in mathematics education have invited teachers to create learning environments that foster students' intellectual development (Fennema & Franke, 1992) by allowing learners to explore mathematical ideas, deepen their understanding of these ideas, and make mathematical connections within and outside of mathematics (Brown & Borko, 1992; National Council of Teachers of Mathematics [NCTM], 1991), and to feel both safe and empowered to do so (Lampert, 1988). Central to these pursuits are various forms of communication, both verbal and written, that allow learners to engage with peers and teachers in the cultivation of rich mathematical knowledge (Hiebert, 1992; Silver & Smith, 1996). As Cobb, Boufi, McClain and Whitenack (1997) have suggested, "The current reform movement in mathematics education places considerable emphasis on the role that classroom discourse can play in supporting students' conceptual development" (p. 258).

Given this emphasis on mathematical communication in the reform literature, it is important for teacher educators to (a) be aware of teachers' conceptions of communication as a vehicle for developing learners' mathematical understanding, and (b) understand how they can help teachers develop practices that foster mathematical communication. This article

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explores these two issues by examining the beliefs, understandings, and instructional practices, as they pertain to mathematical communication, of two beginning teachers.

THEORETICAL PERSPECTIVE

Research has consistently illustrated ways in which teachers "translate their knowledge of mathematics and pedagogy into practice through the filter of their beliefs" (Manouchehri, 1997, p. 198; Thompson, 1992). These beliefs – about mathematics, teaching, and students' learning – are often formed and solidified well before prospective teachers enter preparation programs (Ball, 1988a, 1988b, 1990; Bush, 1986). Cooney (1994) suggested that teacher educators should recognize that their students' beliefs may not be synonymous with those of the preparation program. Hence, because

> changing their [beginning teachers'] beliefs is essential for teachers' development,... it is important to understand not only what teachers believe but also how their beliefs are structured and held (Cooney, Shealy & Arvold, 1998, p. 306).

Given the significance of mathematical communication in reform-based classrooms, it would seem important not only to understand beginning teachers' conceptions of mathematical communication, but also to use the preparation process to orient belief structures of new teachers so that they might create richer learning opportunities for students.

Perspectives on Mathematical Communication

Several interpretations of mathematical communication emerge from reform documents such as the NCTM *Standards* (1989, 1991). For the purpose of this paper, we have chosen to organize the various perspectives into four general categories labeled as uni-directional, contributive, reflective, and instructive communication.

Efforts to reform mathematics classrooms have been motivated in part to address the kind of *uni-directional* communication that is commonplace in our schools. In such settings, teachers tend to dominate discussions by lecturing, asking closed questions, and allowing few opportunities for students to communicate their strategies, ideas, and thinking. Thompson's (1992) review of the literature revealed that many teachers remain wedded to uni-directional approaches in the classroom – instructional strategies that tend to promote mathematics as a static body of knowledge which is first interpreted and conveyed by the teacher, and then passively received by learners. Frykholm (1999) found that, for beginning teachers, expositional teaching is the norm. Nearly 85% of the lessons observed over a three-year period (n = 205) reflected a teacher-centered model in which the dominant form of communication consisted of the teacher talking and students listening. Cooney, Shealy, and Arvold (1998) suggested about uni-directional teaching practices that an "environment dominated by communication of this sort does not encourage students' construction of a broad range of mathematical outcomes as recommended in recent calls for reform in the teaching of mathematics" (p. 306).

Contributive communication focuses on interactions among students and between teacher and students in which the conversation is limited to assistance or sharing, often with little or no deep thought. For example, teachers may provide opportunities for students to discuss mathematical tasks with one another, present solution strategies, or assist each other in the development of solutions and appropriate problem solving strategies. These conversations are typically corrective in nature (e.g., "This is how you do . . . "). Contributive communication might be summarized by drawing on the work of Cobb and colleagues (Cobb et al., 1997), who made note of the informal interactions that occur as students work together on mathematical tasks.

Our definition of *reflective communication* is based on a more complex conception of communication similar to the way Cobb and colleagues (1997) defined "reflective discourse" (p. 258). Reflective communication is like contributive communication in that students share their ideas, strategies, and solutions with peers and teachers. In reflective communication, however, teacher and students use mathematical conversations with each other as springboards for deeper investigations and explorations in which "repeated shifts [occur] such that what the students and teacher do in action subsequently becomes an explicit object of discussion" (Cobb et al., 1997, p. 258). In other words, the authors contended that communication becomes reflective when learners "objectify their prior activity as they [participate] in the discourse" (p. 264). Or, as von Glasersfeld (1991) described it, this type of reflection allows children "to step out of the stream of direct experience, to re-present a chunk of it, and to look at it *as though* [italics original] it were direct experience, while remaining aware of the fact that it is not" (p. 47). This type of reflection does not occur in a vacuum. Students do not "happen to spontaneously begin reflecting at the same moment. Instead, reflection [is] supported and enabled by participation in the discourse" (Cobb et al., 1997, p. 264). Lampert (1990) elaborated on this idea by noting that this kind of rich, reflective discourse often occurs as students attempt to justify or refute conjectures posed by peers.

Instructive communication, the fourth perspective central to this exploration, involves more than interactions between students and teachers. Our notion of instructive communication draws upon Steffe and D'Ambrosio's (1995) suggestion that, "The posing of situations and the encouragement of reflection would remain principal currencies of the teacher. . . . We advocate that situations be posed by teachers to bring forth, sustain and encourage, and *modify* [italics added] the mathematics of students" (p. 157). The act of modification is central to instructive communication in two respects. First, as Steffe and D'Ambrosio suggested, communication of this type can lead to the modification of students' mathematical understanding. In a second sense, however, as the thinking of the students is exposed, teachers not only begin to understand the thought processes, strengths, and limitations of particular students, they also begin to shape subsequent instruction (Fennema & Franke, 1992; Steffe & D'Ambrosio, 1995). It is precisely these student-teacher conversations that actually modify instructional sequences and make this type of communication so powerful. With time and through repeated dialogue, these exchanges "serve as instruments of communication and as a means for the teacher to support and sustain the students' mathematical activity" (Steffe & D'Ambrosio, 1995, p. 158).

Our definitions of these categories are based on the notion that each successive level necessarily assumes the characteristics of its predecessor. For example, if students are communicating reflectively, one can presume that some contributive and uni-directional communication is also taking place. Similarly, in order for classroom communication to reach the instructive level – where the conversations reveal insights about students' thinking that ultimately impact teachers' decisions about future instruction – the discourse must be developed richly enough so as to solicit the kinds of student reflections and insights typically evidenced at preceding levels.

These conceptions of classroom communication point toward the complexities of both examining and implementing communication in the classroom. We develop two case studies of beginning teachers and explore their conceptions and practices related to communication as a method for student comprehension of, and engagement in, secondary level mathematics. The intent of the article is to identify and understand the conceptions these teachers hold and, perhaps more importantly, to highlight the evolution of their thinking and classroom practices related to mathematical communication.

TABLE I

Participants and assignments

Student teacher	University mentor	Cooperating teacher	Teaching assignment
Becky	Dean	Ms. Myler	Algebra I; Pre-calculus
Brad	Joel	Ms. Stevens	Algebra I; Algebra II

METHODOLOGY

Participants

We closely followed two student teachers, each of whom we paired with a university mentor, throughout a five-month student teaching internship. The mentors, both mathematics education doctoral students, volunteered for the study after having expressed a desire to participate in the learning-to-teach process during their tenure as doctoral students. Prior to their graduate school experiences, both mentors had been mathematics teachers in public secondary schools. The preservice teachers also volunteered to participate in the study. They were completing the final requirements for an undergraduate degree in mathematics education. In addition to mathematics content requirements, the students had completed a two-course sequence in mathematics education. The first course focused on the secondary mathematics curriculum, the second course examined reform-based pedagogical strategies appropriate for secondary mathematics instruction. Both courses emphasized the NCTM Standards (1989, 1991). The student teachers were placed in the classroom of veteran teachers, both of whom had more than 8 years of teaching experience.

Several factors led to the selection of the two student teachers from among others who had volunteered to participate in the study. First, both student teachers were placed in the same high school, a factor which facilitated consistency in the data collection procedures, such as weekly observations and interviews. Moreover, within this common school placement the student teachers had similar teaching assignments, which minimized variation in the curriculum they were implementing and the level of students with whom they were working. Further, their cooperating teachers were each well established at the school, endorsed similar teaching philosophies, and had worked in collaboration with the university preparation program on numerous occasions.

Data Sources

In the spirit of qualitative research, we collected and analyzed various data. Data included weekly lesson observations; weekly pre- and post-lesson conferences; a midpoint and a final interview; back-to-campus meetings with a larger group of student teachers; small-group meetings of mentor, classroom teacher, and student teacher; journal entries; and miscellaneous artifacts such as lesson plans and e-mail correspondences. We audiotaped all conferences, interviews, and group meetings.

Methods of Analysis

Erickson (1986, p. 146) suggested that analysis of qualitative data requires one to "generate empirical assertions, largely through induction" and "to establish an evidentiary warrant" for these assertions through a systematic search for confirming and disconfirming data. This process, as Agar (1980, p. 9) has described it, tends to be dialectic as opposed to linear. Specifically, it requires an iterative and systematic fracturing of the data that leads to generative questions and, ultimately, a discovery of core categories, themes, and "regularities in the data" (Wolcott, 1993, p. 33). Thus, in our search for core themes in the data, the entire data set was examined repeatedly and thoroughly. Based largely on the recommendations of Wolcott (1993), Strauss (1987), and Spradley (1979), we applied analytical tools that included expansion of field notes, writing memos, and coding the data.

Expansion of field notes. Spradley (1979) emphasized the importance of building on original field notes. "As soon as possible after each field session [the researcher] should fill in details and record things that were not recorded on the spot" (p. 75). Based on his recommendations, we immediately expanded (reworked, filled in, and completed) field notes taken during observations, conferences, interviews, and seminar sessions, so as to retain a more accurate and complete description of the field experience.

Writing memos. Throughout the data collection and analysis, we gave particular attention to Strauss' (1987) suggestion to include a "memoing" process for the purpose of capturing insights, questions, and understandings as they occurred throughout the research. Following his recommendations, we kept a separate file for random thoughts, insights, ideas, or possible interpretations of the data as they were both collected and analyzed.

Coding the data. Strauss (1987) recommended a coding process in the early stages of the analysis that "follows upon and leads to generative questions,... fractures the data, thus freeing the researcher from description and forcing interpretation to higher levels of abstraction ... [and] is the pivotal operation for moving toward the discovery of a core category or categories" (p. 55). Through a process of carefully reading transcripts, field notes, and observational records, we explored and noted regularities and patterns in the data. We then organized and labeled these themes as either external codes (larger, theoretical concepts in the data), or internal codes (particular themes within an external code). For example, one of the primary themes that emerged and served as an external code involved the relationship between classroom discourse and classroom management. As both student teachers at times opened their classrooms for discussion, they found themselves challenged with issues of management. Under this umbrella of classroom management (the external code), the student teachers spoke of issues of control, constructive student participation, and time constraints (internal codes).

Analysis. We first analyzed the data sets by case. That is, at the conclusion of the student teaching experience we examined the entire data set for each student teacher, independently of data from the other case study. In so doing, we followed Erickson's (1986) suggestion that initial stages in comparative analysis should consist first of "patterns of generalization within the case at hand, rather than generalizations from one case or setting to another" (p. 148). Upon completion of an initial analysis of the data sets for each student teacher, we identified common themes that emerged across the two cases. For example, for each of the student teachers, a primary impediment to enhancing communication in the classroom was the perceived constraint that they had to maintain adequate pace and content coverage. We aggregated the codes from the two independent data sets that related to content coverage into one file, in which we completed further analysis across the cases. Part of this subsequent analysis entailed the organization of excerpts from the data into domains or classes (Schatzman & Strauss, 1973) in which we drew relationships, where possible, between themes and across data sets. This process of establishing domain analyses (Spradley, 1979) helped organize the emerging themes across the cases which ultimately led to the primary assertions for this report.

FINDINGS AND INTERPRETATIONS

We present the case studies of the two student teachers, which provide the context for a subsequent discussion of the implications of this research for mathematics education and teacher preparation. The presentation of the findings is organized around three primary themes: (a) the students' dispositions toward and definitions of mathematical communication as expressed at the onset of the study; (b) the experiences, practices, and reflections that took place during the student teaching experience as they related to mathematical communication; and (c) the student teachers' conceptions of communication as evidenced at or near the conclusion of the student teaching experience.

Case 1: Becky

Initial dispositions and definitions. Before her student teaching began, Becky was asked to reflect on mathematics and mathematics teaching. Although the assignment did not include specific instructions to address the topic of mathematical communication, there were hints in Becky's writing that suggested she recognized the need for students to communicate with one another. As she was describing the ideal mathematics classroom, Becky noted that students "should be working together" on math problems in order "to help each other understand the material better." Although Becky may have intended the comment to reflect directly her belief that a mathematics classroom should be "inviting and positive" for students, as she said elsewhere, her words do nevertheless suggest that she was at least in part oriented toward the notion of contributive communication – students sharing ideas.

In her first conference with Dean, her university mentor, Becky admitted that she still had many unresolved questions and uncertainties about the classroom. After her first week of assuming full control of her classes, she noted how the demands of teaching had left her "wishy-washy" regarding her classroom structure. As she confessed to Dean, "I had no clue what I was doing. It was like making decisions at random and changing them later" (Post-Lesson Conference 1, 10/6/95).

Despite her many questions, evidence existed to suggest that Becky maintained some certainty that her students should be talking with one another. As she and Dean continued their post-lesson conversation, Becky identified several issues of frustration that pointed toward her use of both uni-directional and contributive approaches to communication. For example, like many beginning teachers, Becky was concerned about classroom management. She identified management issues that would become the seeds for her explorations with various instructional formats that greatly impacted the classroom discourse.

One of her concerns had to do with how her students' varying abilities and mathematical understandings complicated her attempts to manage the class, particularly as students worked in pairs.

> There are so many different levels of ability in the class that I have a really hard time keeping them all on what I am talking about. Like, on number one, some of them [pairs] were off solving it another way while I was still trying to work out the brackets. They are too vocal.... When they are asking questions and I start explaining to them, they are not paying attention again because it is someone else's confusion that I am answering, and not theirs (Post-Lesson Conference 1, 10/6/95).

Becky appeared concerned about managing the conversations that occurred when students were given opportunity to work together. More specifically, this quote points to Becky's struggle to find a balance between the first two forms of communication. On the one hand, she maintained her commitment to contributive communication – that students should be talking about mathematics; on the other hand, however, her decision to "start explaining" in response to student questions was an indication of uni-directional instruction. Early in the semester, Becky valued opportunities for students to communicate with each other, but she also felt a need to control the discussions. The following excerpt lends further support to Becky's dual-perspective: "They [students] don't really think that they need to hear what I have to say in class. So, they are constantly working, and now I am torn about splitting them up. Because it is a good thing that they are working together, but it is distracting" (Post-Lesson Conference 1, 10/6/95). It appeared that Becky was reconciling her initial commitment to contributive communication with the strong pull she felt to communicate directly about the mathematical ideas she felt were most important.

In summary, it appeared that Becky initially affirmed the idea that students could benefit from talking with one another in the classroom. The management issue that arose in doing so, however, presented her with various dilemmas. How could she allow students to communicate with each other while at the same time maintaining control? How should she account for the variety of conversations that emerged due to students' differing levels of abilities and understandings? How could she keep a small group of students from dominating larger class discussions? As Becky summarized her frustrations at the end of the first conference, she noted, "I just hate being at war. I hate to be on opposing ends of the rope, and I am always having to pull them harder" (Post-Lesson Conference 1). Her notions of "being at war" and having to "pull them harder" suggested that she was being challenged by what she felt were two seemingly opposing communication forms.

Experiences, classroom practices, and reflections. The first evidence of change in Becky's use of communication grew directly from the dilemma mentioned previously. In order to keep students on task and to avoid situations in which only a few students responded to typically convergent questions, Becky sought Dean's help in developing questions that would elicit deeper thinking and promote more involvement and student interaction.

> One thing that I am curious about, and that I am trying to be aware of is my questions. The kind of questions that I am asking in class is one thing that I would really like you to watch for next time. Because, I think a lot of times I ask things where they don't understand what I want them to tell me. And, I don't always ask one person, so they are all kind of looking at me like, "What do you want?" (Post-Lesson Conference 2, 10/13/95).

Becky's request for help in developing her questioning strategies suggested that she wanted to promote a different type of thinking and interaction in her classroom. Yet, her remark that students "don't know what I want them to tell me" suggests that Becky had not yet moved to a reflective level in her instruction. Although she wanted her students to communicate, she appeared to have clear ideas in mind as to the appropriate nature and direction of the discourse.

As a result of her interest in using questions more strategically, Dean and Becky discussed an approach that, if implemented, would have pushed Becky toward reflective communication. Dean began by suggesting that teachers might "throw out a question – a focused question that isn't obvious and really takes some thought. In some cases, the question should even be debatable". Becky responded to this idea by suggesting, "Yes. They could then discuss it with a partner. If they give an answer, then I can make them verify it against each other" (Post-Lesson Conference 2, 10/13/95). Although there is some hint in this quote that Becky still viewed herself as the authority ("I can make them . . . "), this comment represented the first instance in which Becky sought to solicit deeper mathematical communication from her students by having them engage in reflective communication.

During the next classroom observation, both Dean and Becky noted the improvements in students' participation and in the quality of their discussions. Dean attributed the growth to Becky's detailed focus on asking divergent questions that elicited deeper exchanges between students. To provide an example of the kinds of conversations that took place in Becky's classroom, we reconstructed the following excerpts from observational field notes. The excerpts not only provide a glimpse into the ways in which Becky was encouraging her students to communicate, but also point toward the ways in which Becky was introducing and exploring mathematical content with her students.

The topic under investigation was the exploration of the volume of a box to be constructed by folding an 8.5-by-11-inch rectangular piece of paper with cut-out corners. In particular, Becky was asking her students to think about an algebraic representation of the volume of the open box as it related to the size of the cut-out corners.

Becky: Okay guys. In your groups, discuss what this diagram on the board is telling you. How could you use this diagram to think about the volume of a box?
Response: The students shared ideas for several minutes at their tables, while Becky wrote “volume =?” under a diagram of the rectangular paper, using dotted lines to represent the corner cutouts. She then allowed students to summarize the content of their group conversations for the whole class. After the class agreed that the box could be made by folding the paper along the dotted lines, Becky continued:
Becky: So, what if I changed the length of the dotted lines in the corners? What would happen if I folded up the box then? In your groups, hypothesize what would happen to the volume if you were to change the size of the cutouts in the corner.
Response: After several minutes of discussion, some groups suggested that the bigger the cut-out, the greater the volume. Other groups suggested that the smaller the cutout, the greater the volume. Becky responded:
Becky: Okay, then, since we have some contradicting ideas, let’s actually do some samples. In your groups, pick a number for x and actually figure out what the volume would be. How do we find the volume of a box again?
Response: After a brief discussion about the formula for volume, the students then computed the volume of their boxes with a specific value for x. Becky led a discussion of the critical values for x by asking the following questions:
Becky: Can x be any number? Are there some numbers that will not work for x? Why is that?
Response: Becky then led a discussion in which students, after talking in their groups, determined that the length of x could not be greater than half the width of the paper, or else there would be no box. She then helped the students develop a generic formula for the volume of the box: Volume = $(8.5 - 2x)(11 - 2x)(x)$. After working through several examples with the formula, Becky turned to the use of calculators to further explore the context (Excerpts taken from field notes, 10/19/95).

Although not a complete account of the class period, the excerpts provide insights into the communication forms Becky was using in the classroom. Moreover, they provide insight into the ways in which Becky believed she should introduce her students to new mathematical concepts and ideas. As he told Becky after class, Dean noted these elements in Becky’s teaching, particularly the ways in which her questions led to deeper conversations about the mathematical content at hand.

Really good questions, Becky. . . . You really helped them think about not just an equation, but how to analyze the problem and use the calculator as a tool and not as a magic wand to answer all their questions . . . how they needed to look for restrictions for *x*, estimate the volume at a certain point, find the max volume (Post-Lesson Conference 3, 10/19/95).

Moreover, Dean praised Becky for allowing a student to come to the board during the lesson to present his solution strategy to the class, citing it as an excellent opportunity for the student to communicate and "model his thought process." In a journal entry, Becky also described the improvements she felt she was making in helping students engage in and verbalize the mathematics at hand.

I am already getting better at pulling them through with questions even when they don't know the answer. I have found some success with some students, and now they are like, "Wait a minute, I don't get that." Before, they wouldn't have even asked. They wouldn't have cared enough to ask (Journal entry, 10/18/95).

Notable about Becky's progress was the extent to which her steps toward fostering mathematical communication were at odds with the beliefs and practices of her cooperating teacher. The following exchange is one of several that occurred over the course of the semester that indicated a gap between the perspectives of Dean and Ms. Myler. As they were summarizing a lesson during Dean's fourth classroom visit, the discussion turned toward questioning strategies.

Dean: I thought the open-endedness of your questions was great. Right after you asked your first question, I thought, 'Wow, this is going to be interesting.' And the kids gave great answers! So, your choice of questions was good.
Ms. Myler: But, sometimes I was wondering why you asked particular questions, and what you were hoping to get from them. Some of them had so many potential answers. I was pretty critical at first (Post-Lesson Conference 4, 10/25/95).

Becky defended her questioning strategies by noting the degree to which students had responded. Attributing their responses to other factors, however, Ms. Myler responded,

I think the students answered the questions because they wanted to help you out. They know you are trying to teach the lesson that way, and they know the routine. They want to contribute because they like to show other people what they know (Post-Lesson Conference 4, 10/25/95).

Whereas Dean praised Becky for her divergent questions and the student interactions they fostered, Ms. Myler appeared to criticize the fact that there was not one readily obvious answer to the questions. Moreover, she suggested that students were participating not because of the insights they were gaining, but rather to please Becky or to draw attention to themselves.

We share this example to illustrate the strength of Becky's emerging disposition toward student engagement and interaction in the classroom.

Given what is known in the research literature about the powerful influence that cooperating teachers have on beginning teachers (e.g., Brown & Borko, 1992; Zeichner & Gore, 1990), it was notable that Becky continued to push her classroom practices toward student interaction despite the reservations of her cooperating teacher.

Other evidence existed as well to suggest that Becky was experimenting with various instructional formats that were intended to foster classroom communication. As noted in Dean's observational field notes, for example, she regularly had students share their work at the board or on the overhead, created cooperative learning activities, had students write about their mathematical processes, and continued to work on improving large-group discussions. As Becky continued to experiment with these instructional formats, she became less comfortable with the traditional stereotype of the teacher as the "knowing authority" in the room. Becky began to resist teacher centered, or uni-directional, methods of instruction.

> I feel so uncomfortable teaching this way – having to say things like, "You should already know how to do this." It makes me feel like an all-knowing brain that just holds all this stuff in my head. It is like, "No. It isn't true for me either. I am not a mathematical genius." I can relate to their frustrations when classes are taught that way (Post-Lesson Conference 8, 12/1/95).

As evidence of this resistance to direct teaching methods, Becky often structured group activities. "I like putting them in small groups and making someone who isn't doing well on the math be in charge of the group. . . . In groups, everyone is responsible for the answer. If somebody doesn t know it, then you are the teacher. You are responsible for them" (Post-Lesson Conference 7, 11/15/95). It was also not uncommon for her to have students work together at the board to communicate their solution strategies. "I do that about once a week now – have volunteers teach at the board. . . . Having them go to the board is another way for me to kind of, on the spot, evaluate where the class is. By watching them do it, I am not just blabbing" (Post-Lesson Conference 8, 12/1/95). Notable about this quote is Becky's recognition that having students articulate their thinking was an effective formative assessment tool – a fundamental element of reflective and instructive communication. Dean also recognized and praised Becky's progress.

> This is a different class than the one I saw in here a month ago. You have really done a great job with them. . . . Even the folks who usually don't participate were talking about it. Everyone was really getting involved in the problem. It was good. Students were enthusiastic about it, it was a fruitful discussion, and there were multiple methods in solving the problems (Post-Lesson Conference 10, 12/8/95).

Final dispositions toward mathematical communication. In the final interview, Dean confirmed Becky's growth as he described the typical day in her classroom, noting in particular the mathematical questions she used to motivate both the conversation and the conceptualization of the mathematics.

Becky provides a rich mathematics experience for her students.... Student group work and labs are initiated, data is collected, and the results are inputted. As students record their group's data on the board, a large group discussion begins. One student volunteers to demonstrate how the collective data is graphed, Becky encourages the students to dig deeper in their analysis, asking them critical questions: 'What if we used dice instead of coins? Would the results change if we used more coins? Less coins? When is the probability of an event a good predictor?' As the processing of the activity winds down, Becky asks the students to respond to some writing prompts before the end of the period (Final Interview, 1/22/96).

Dean's description indicates that Becky was making instructional decisions that promoted rich communication in the classroom – certainly beyond the contributive and uni-directional notions of communication she brought to the preparation experience. It also points toward the ways in which these rich forums for communication contributed to the students' understandings of mathematics as they became heavily invested and involved in the conversations and contexts at hand. Certainly, we are not suggesting that Becky abandoned all direct forms of communication in the classroom. Indeed, the data indicate evidence across multiple sources that Becky used a variety of techniques to converse with her students. It was certainly evident, however, that she had become comfortable promoting and implementing reflective communication. Whether she continued to build her strategies to the point of instructive communication is, of course, difficult to know. There were, however, hints that Becky was moving in this direction as she was seeking to use the input and comments of her students for the planning and delivery of future instruction.

Case 2: Brad

Initial dispositions and definitions. Early in the semester, Brad'spoke knowledgeably about the tenets of reform. In initial writing exercises and conversations, he reported that the NCTM Standards promoted the integration of technology into mathematics classrooms, contextually based problems, the creation of meaning, exploratory learning, high-level thinking, and connections within and outside of mathematics. Brad initially described his goals for his students' learning by stating,

I don't want them to just memorize the graphs of stuff even though I said they could get away with it even if they don't understand it. I mean, that would be one way to get it.

But, I want them to understand why – you know, why is $y = x$ squared a parabola? From a distance aspect – why can't distance be negative? That type of thing. Where you actually understand what the mathematics means (Post-Lesson Conference 1, 9/25/95).

This quote suggests that Brad, at the beginning of his student-teaching experience, was interested in helping his students cultivate rich mathematical understanding. Although Brad had not yet spoken explicitly about the role of mathematical communication in this process, he did discuss instructional strategies that were consistent with the goals of reform-based mathematics instruction.

As initial conversations between Brad and Joel, his university mentor, continued they discussed issues that hinted at Brad's dispositions toward and understandings of mathematical communication. When Joel asked Brad to suggest various methods through which a teacher might better understand students' mathematical knowledge, Brad did not acknowledge the possibility of having students communicate with one another or the teacher. Brad stated that there was "really no way to measure" the quality of students' thinking aside from quizzes and tests. When Joel asked Brad about informal assessment involving classroom interactions as a way to gauge his students' understanding, Brad responded by saying, "Really, you can't. I mean, if they don't raise their hand when they don't understand, then you will never know" (Post-Lesson Conference 1, 9/25/95).

When Joel asked Brad how he felt about having students communicate their thinking or strategies to the rest of the class. Brad responded,

I haven't really tried that in this class yet, or any other classes. . . . It is really not something that [my cooperating teacher] does. And that doesn't mean that I can't do it, but I am trying to stay parallel to her (Post Lesson Conference 1, 9/25/95).

This conversation suggested several notable factors about Brad's initial disposition toward communication in the classroom. Brad was able to articulate several of the key principles underlying the NCTM *Standards* (1989) in general. Yet, it appeared as though he did not have a deeper understanding of how those principles might be related to the notion of mathematical communication or how they might emerge in the context of the classroom. There was no evidence suggesting an understanding of various types of communication. Moreover, foreshadowing what would be a significant factor in his experience, Brad's early deference to the preferences and teaching strategies of his cooperating teacher was notable.

Experiences, teaching practices, and reflections. Despite the fact that Brad voiced his interest in fostering rich understanding, the first lessons Joel observed did not appear to reflect his previously stated commitment to the visions of the reform movement. Although Brad reported that he did not want students simply to memorize facts but rather to understand mathematics conceptually and to experience mathematics actively, there was little evidence of these goals in Brad's teaching. During the first lesson observation, for example, Joel's field notes indicated that throughout a 20 minute lecture Brad asked his class only three questions, none of which was open-ended (Field Notes, Observation 1). Each of the questions he asked was answered with either "yes" or "no" by his students. During the post-lesson conference Brad, in support of his uni-directional teaching methods, described the role of the teacher as one of "telling." He noted about his own teaching,

> I would like to show them how to do things. . . . I think teaching is showing them how to do it and catching their interest and give [*sic*] them some time to struggle with it and being there after they have struggled to show them what went wrong and to try to clear up what they don't get (Post-Lesson Conference 2, 10/2/95).

This quote suggests that Brad believed it was the teacher's job to show learners how to solve mathematical problems and to be there to answer questions that might arise. Perhaps this approach was due in part to his own experiences as a learner. In his mathematical autobiography, Brad recalled from his own experiences that he liked it when his teachers demonstrated solution strategies for the class, followed by a number of similar problems to be worked by students. Although he admitted that not all students learned best this way, he nevertheless thought that it was the best way to teach. In short, Brad advocated uni-directional communication, a position that was confirmed as Joel continued the classroom observations.

During the next observation, Joel reported his intent to pay special attention to developments in Brad's use of communication strategies, particularly how they impacted his students' engagement and understanding of the mathematics. In general, Joel's observational notes showed that Brad's practices went unchanged. We share one example from the class period in which it was clear that Brad's tendency toward direct instruction significantly limited opportunities for his students to further deepen their understanding of linear equations. Joel recorded in his field notes that, at one point early in the lesson on developing understanding of various algebraic representations of linear equations, a student asked the question, "Why is 3x + 27 = 0 not already in the y-intercept form?" Instead of taking the opportunity to further the investigation by posing the question to the class or engaging the student, Brad quickly exclaimed, "Because there is no *y* in it" (Lesson Observation 3, 10/9/95), and then immediately moved on to an additional example.

In the post-lesson conference, Brad commented on this interaction. He admitted that he could have developed the issue further or perhaps could have allowed other students to discuss the question, but he chose a different course of action based on three reasons. First, he stated that it was much “easier to tell the student what to do or what the answer is than to have other students explain it” (Post-Lesson Conference 3, 10/9/95). Throughout the term Brad made many similar comments, including one several weeks later when Brad was convinced that it was not necessary for him to help his students understand the process of factoring perfect square trinomials. He stated, “I wouldn’t show them algebraically. I would just *tell* them: ‘because of the pattern, divide by two and square it.’ And those who can handle why it is will see it” (Post-Lesson Conference 7, 11/13/95). Second, not only did Brad feel that it was an easier method of teaching, he believed that *telling* his students the answer allowed him to address a second concern, that is, keeping pace with his cooperating teacher’s classes. As he noted on several occasions, “taking the time to do that would have put me behind our schedule for things” (Post-Lesson Conference 3, 10/9/95). Third, Brad was also concerned that, by opening the discussion up to other students, he was running the risk of having those students “explain things the wrong way” (Post-Lesson Conference 3, 10/9/95). This sentiment emerged repeatedly from the data, one example being a conference two weeks later in which Brad expressed his concern that students, if given the chance, would “go off and explain something else that I did not want explained, and I [know] the other students don’t really want to hear it either” (Post-Lesson Conference 5, 10/23/95). At the conclusion of the third classroom visitation and conference, Joel recorded that Brad did not appear to value forms of communication other than those that were teacher directed.

Throughout the fourth and fifth observations, Joel continued to record each question Brad asked as well as the kind of conversation the question elicited. Although Brad raised his average from three to seven questions per lesson, the questions themselves continued to be mostly convergent and lower level as students normally responded with one- and two-word answers. Joel noted one occasion, however, in which Brad asked a question that might have led to a significant conversation, “How did you get f?” (Lesson Observation 5, 10/23/95). After pausing for no more than a few seconds, Brad answered the question with a full explanation. Later, Joel asked him to reflect on his quick reaction and verbal explanation. Brad responded by saying that asking questions that require students to communicate “is difficult, especially when the class is not very verbal” (Post-Lesson Conference 5, 10/23/95). Moreover,

> I was thinking about time constraints. If I sit around and wait for them to answer this question, I am not going to get through what I need to. And, sometimes when I am up

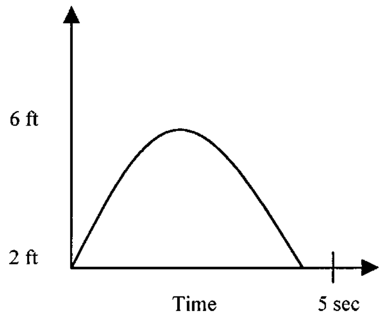
there, I know ... I can see you back there writing 'wait time' or something! ... I guess I could have used my brain and thought before I asked the question, 'Am I going to give them ample time to answer this?' That might be something I should do rather than asking questions and answering them right away because that reinforces a bad behavior (Post-Lesson Conference 5, 10/23/95).

As we revisited initial codes from classroom observational notes and interview transcriptions, this particular quote best illustrated a significant dilemma with which Brad seemed to be wrestling. He was aware that he should allow more time for discussion, but he clearly felt the need to push ahead so that there would be ample time to cover the material. In fact, as illustrated in the previous excerpt, due to this perceived time constraint it appeared as though Brad believed he should not have asked the question in the first place.

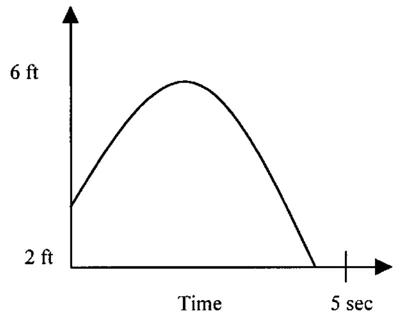
At the conclusion of the fifth lesson observation and conference, Joel and Brad agreed to plan a lesson together that would involve students in meaningful discussion and interaction. They brainstormed specific questions students could explore in ways that would strengthen their understanding of the concept of graphing quadratics. Also built into the lesson was an opportunity for four students with different solution strategies to explain their ideas to the rest of the class. Brad would use the sample work of these students to practice asking follow-up questions that would promote greater understanding of the problem. Brad was eager to implement the plan, anticipating the opportunity to initiate a discussion that would foster reflection and student interaction.

In the lesson, students were asked to produce an equation that expressed the trajectory of an eraser Brad tossed into the air. The key mathematical concepts for the lesson included: drawing the trajectory of the tossed eraser, producing an equation that fit the trajectory, and discussing the key elements in the equation that best reflected the path. After several minutes, Brad asked four students to present their work to the class. Inherent in these four solutions were several distinctly different approaches to the problem, two of which appear in Figure 1. In succession, each student went to the board, read his or her equation, and sat down without further comment. Joel's field notes indicated that Brad did not ask a single question of the students, thereby failing not only to elicit further communication and evidence of student understanding or reasoning, but also failing to implement what he had planned with Joel. This is particularly significant given the rich discussion that might have taken place had Brad carefully explored the conceptual differences between the two solutions in Figure 1 (Lesson Observation 7, 11/13/95).

After the lesson, Brad and Joel discussed these two situations in particular. Joel began by asking Brad how he felt about the portion of the



Student 1 drew the above graph and described it as $y = -(x - 2)^2 + 4$, where the eraser started at about 2 feet and ended at the same height.



Student 2 drew the above graph and described it as $y = -(x - 2)^2 + 6$, where the eraser began at about 2 feet but ended at 0 feet (on the floor).

Figure 1. Two students responses to the tossed-eraser problem.

lesson that involved the volunteers. Brad reported that he “didn’t like it because the other students didn t have anything else to do at the time. Some of the students were goofing off” (Post-Lesson Conference 7, 11/13/95). Moreover, Brad continued,

I wanted to do the work because I wanted to get through the problems faster. I didn’t want to write it [students’ solutions] down and explain every one. So I thought if they put them up there, then I could just go through and explain them quickly. But, unfortunately, some of the people who came up did not show their work, so I had to go through and do the work myself. I guess I was a little disappointed in them in that. Maybe I just didn’t get from them what I expected.

Joel asked Brad if he could think of any other ways in which he could have handled the situation.

Brad: I don’t know. I wonder if on the problem that they ask for, if it would be a good idea to have everyone at a table pull out a sheet of paper and do those problems and turn it in. They could help each other.
Joel: What if you had had the students stay up there and had them explain it themselves?
Brad: I thought of doing that. But, when I looked at the work, I didn’t know if I wanted them to explain it because, for example, the first one only had one line down. No picture, no nothing. The second one was a definition. And the third one I sort of wanted to talk about because it was that distance stuff. Something that we talked about in the reading today that I did not necessarily cover. So, maybe I just wanted to talk about that a second (Post-Lesson Conference 7, 11/13/95).

It appeared as though Brad had not moved significantly from where he was earlier in the semester. Brad continued to believe that students could not articulate the mathematics as well as he could, which would, therefore, impede the progress of the lesson if they were allowed to do so. The pace of the lesson was important due to the pressure of maintaining the timeline for content coverage set by his cooperating teacher. Moreover, Brad appeared to remain wedded to the uni-directional belief that students learned best by watching examples and listening to explanations, as opposed to exploring their questions in dialogue with one another.

Final dispositions toward mathematical communication. Brad began the year by stating that mathematical communication was important and that students should explain their thinking and reasoning. Yet, throughout the semester his students had few opportunities to discuss ideas and strategies with one another as they grappled with their understanding of a mathematical concept. Moreover, it appeared as though Brad did not see mathematical communication as a method to assist him in monitoring students' understanding. Despite what he said early in the semester, Brad's beliefs about classroom communication were fairly fixed and matched well with his instructional practices. He rarely implemented teaching practices that encouraged students to discuss the mathematics, to reflect on key mathematical ideas by sharing their own ideas and strategies, or to describe their thinking in order to monitor and adapt his instructional methods. The consistency in Brad's thinking and actions suggests that his perceptions of mathematical communication were fairly narrow and bore little resemblance to the reform movement that Brad talked about early in the semester. Brad began his student-teaching semester with beliefs that were consistent with a uni-directional approach to mathematics instruction, and it appeared as though his belief structure changed little over the course of the semester.

DISCUSSION

The case studies show an interesting contrast between two beginning teachers and their ability and willingness to implement various forms of communication in the classroom. Becky appreciated the importance of facilitating opportunities for her students to share ideas. In contrast, Brad remained convinced that teacher directed (uni-directional) instruction was the most effective means to student learning. Although there was no intent to rigorously control this study, Becky and Brad were the same age, had similar content preparation, participated together in two method courses, taught in the same school with nearly identical schedules, had remarkably similar cooperating teachers, and received supervision from university mentors who each approached mathematics education from a committed, reform-based perspective. These similarities give rise to perhaps the most persistent and important questions to emerge from this study: *Why* did these two preservice teachers develop as they did? What might have caused one teacher to shun richer communication strategies, whereas the other appeared to grow significantly in her willingness and ability to involve her students in various forms of conversation and collaboration?

The Role of Beliefs and Dispositions

We know that beginning teachers filter their preparation experiences through their beliefs and that "novice mathematics teachers that have been exposed to years of traditional instruction may be socialized to this manner of teaching; therefore, it may be very difficult to . . . help them develop different conceptualizations of mathematics teaching" (Brown & Borko, 1992, p. 222).

These assertions provide one possible explanation for Brad's development (or lack thereof) throughout the study. Brad brought to the student teaching experience traditional beliefs about the roles of teachers and students, as well as the nature of mathematics – a static body of rules and procedures that needed to be conveyed to learners by the teacher. Moreover, he came to the mathematics classroom with a commitment that was somewhat suspect. By his own admission, Brad reported that his mathematical experiences as a student had not been overly positive and that his mathematical understanding was not as strong as his high school and college grades might indicate (Interview, 11/1/95). Brad noted that he became a teacher of mathematics somewhat by default. Although he reported liking the idea of teaching in general: "I like kids, and I like teaching," he made his decision to teach mathematics simply because "math seemed to be the best way to get a job" (Group Meeting, 10/2/95). Hence, we see a picture emerging in which Brad came to the mathematics classroom not only with some hesitation, but also with a natural inclination toward uni-directional instruction. As these initial dispositions were reinforced by Ms. Stevens early in the semester, Brad fell into routines and a way of thinking that would not be shaken.

What was it about Becky's beliefs, however, that allowed her to break from the traditional instructional mindset that she both experienced as a learner and was encouraged to follow as a student teacher? We would like to suggest that Becky had a disposition toward reflection that Brad did not possess, as well as broader vision and beliefs about the nature of mathematics. Throughout the data, evidence existed to suggest that Becky believed the study of mathematics needed to be an active endeavor. As shared earlier, Becky was uncomfortable with the idea that she, the teacher, was the primary owner of the mathematical knowledge in the room. Rather, it appeared as though Becky approached the classroom attentive to the idea that students needed to be actively involved if meaningful learning was to take place. In an interview, she explained that the origin of this belief could be found in her reaction against the instruction she had received in college. "I felt like I had something to contribute. But, in my college math classes, we didn't talk. We just sat there and had things spooned to us. That made me mad" (Midpoint Interview, 10/31/95). One of the primary distinctions between Becky and Brad, then, was the degree to which their beliefs about the nature of mathematics, as well as the processes whereby mathematics should be taught and learned, impacted their development as beginning teachers.

Hence, this study appears to highlight the important relationship between teachers' requisite beliefs about the nature of mathematics teaching and learning, and their ability to move along the communication continuum advocated in this article. As Brad's beliefs about uni-directional teaching were confirmed and strengthened by his cooperating teacher, even a significant intrusion by his university mentor was insufficient for Brad to implement beginning steps toward contributive or reflective communication. In contrast, Becky's conceptions about mathematics, its teaching and learning contributed to her natural evolution toward fostering reflective communication in the classroom.

The Role of Mathematical Content Knowledge

Although we had not intended to make mathematical content knowledge a primary focus of this study, it became clear to us that for both Brad and Becky content knowledge played a role in the kinds of communication strategies they implemented in the classroom. On the one hand, Brad appeared to be very hesitant to deviate from the standard, traditional procedures and definitions of mathematics that he had received as a learner. In a mathematical autobiography, Brad admitted that his content knowledge was weaker than he would have liked and that he had struggled in his college courses. We have also shared excerpts from the data that show that Brad, when challenged with content questions from his students, rarely if ever explored those questions in the depth that they deserved. One might infer that Brad's hesitancy to do so was in part linked to a lack of confidence in his mathematical knowledge.

Although Becky also admitted that she had holes in her mathematical knowledge, it did not appear as though it impacted her teaching. On the contrary, Becky regularly held discussions that not only depended on her ability to use her content knowledge to navigate and lead the conversation, but also challenged her own content knowledge as well. Whereas Brad did not want to promote times of uncertainty and dialogue in his classroom, the data presented earlier suggest that Becky even tried to teach toward those moments of uncertainty by fostering rich conversations and dialogue. Certainly, her confidence in her own mathematical knowledge contributed to, if not facilitated, this kind of reflective communication.

The Role of Teacher Socialization

One widely studied aspect of the learning-to-teach process is the role of the cooperating teacher on the developing thoughts and practices of preservice teachers (e.g., Brown & Borko, 1992; Zeichner & Gore, 1990). Recent research suggests that cooperating teachers influence preservice teachers' thinking, beliefs, and classroom practices more than any other factor throughout the professional preparation program (Frykholm, 1996, 1999). What seems to have emerged from this study is an interesting contrast. On the one hand, Brad fits the profile described in research studies detailing the impact of cooperating teachers and socialization factors upon the development of beginning teachers. Despite reform-based preparation coursework and university support, Brad felt comfortable mirroring the practices of his cooperating teacher. Brad and Ms. Stevens planned lessons that promoted teacher-centered instruction and allowed Brad to stay well within his comfort zone. He accepted Ms. Stevens' assumptions about classroom communication and therefore never attempted to branch from their parallel teaching. As Joel reflected late in the semester, "Brad ended up more traditional at the end than when he started" (personal communication, 1/5/96).

Becky's development, on the other hand, seems to contradict the same body of research findings. Despite the pressures she faced from her cooperating teacher to conform to specific content and pedagogical guidelines, Becky nevertheless made significant strides in pushing her own beliefs and practices in a direction that was, in many ways, opposed to that of her cooperating teacher. At the beginning of the semester, Becky was mindful of Ms. Myler's preference for and model of direct instruction. In fact, Becky reported early on that Ms. Myler was mechanical in the way that she was "concentrating on the details of the lesson, the content, what I would say" (Final Interview, 1/28/96). Yet, despite this starting place and modeling from her cooperating teacher, the data suggest that Becky was predisposed to think critically about the educational implications of various forms of communication. She was able to broaden her teaching strategies to include both contributive and reflective forms of communication. As she noted at the end of the semester, she had shifted her focus from what *she* should be doing, to how her *students* might be thinking and interacting. As she noted,

At the beginning, I was really concentrating on the details for the lesson, the content, what I would say, what the concepts were – just really detailed in the mechanics of it. Toward the end of the semester, I became more aware of the *process* of learning. That became just as important as what I was going to be teaching. At first, it was like, 'What examples can *I* give?' Toward the end, it wasn't so much the examples themselves that I was interested in, but how they [students] were thinking about them and interpreting them (Final Interview, 1/28/96).

CONCLUSIONS AND IMPLICATIONS

Revisiting the Communication Constructs

The four constructs of uni-directional, contributive, reflective, and instructive communication were helpful in describing classroom practice, particularly as it developed beyond uni-directional communication to more complex forms of instruction. As we stated at the onset of this article, we developed these constructs to be hierarchical in nature. The study confirmed that idea and also led us to think more deeply about the relationships and overlaps between and among the levels of communication. Specifically, the continuum we are advocating begins with teacher centered, expositional, and direct instruction as illustrated by the teaching documented for Brad. Teachers who begin to move beyond the uni-directional instructional format begin to encourage students to share their mathematical ideas, solutions, and insights. In this sense, the students are *contributing* to the classroom discourse although not yet fundamentally altering it.

The next step – from contributive to reflective communication – seems to be a critical one in that for the first time the sharing of students' ideas and insights is done with the intent of deepening the mathematical understanding of the participants. At this third level of communication, teachers provide opportunities for students to reflect on the relationships within the mathematical topics by focusing on other students' and the teacher's ideas, insights, and strategies. The fourth level of instructive communication is one in which the course of the classroom experience is altered as a result of the conversation(s) in the classroom. Specifically, at this level a teacher's practice and instructional decisions are so entwined with students' mathematical ideas and propositions that the path of the classroom progression is altered in order to build upon and deepen students' present understanding of the mathematics at hand.

As others have noted, the pattern of communication in the first two levels might be described as univocal (Peressini & Knuth, 1998; Wertsch & Toma, 1995). In both of these initial levels, the conversations are predominantly focused on explaining and telling, as knowledge seems to be transmitted from one person to another. How this knowledge is conveyed, however, becomes important. That is, the difference between uni-directional and contributive communication may be characterized by the social norms of the classroom (Cobb, Wood & Yackel, 1993; Kazemi, 1998; Kazemi & Stipek, 1997). Uni-directional communication assumes that the teacher is the authority of mathematical knowledge and explains it to students. At the contributive level, despite the fact that the teacher still retains the mathematical authority, students are allowed to articulate solution strategies. There are, therefore, important distinctions between these first two levels that depend on the social norms of the classroom.

The third and fourth communication levels – reflective and instructive – are conceptually different from the first two levels. Again, as others have noted, in these types of communication the conversation shifts from a focus on transmitting information to generating meaning or dialogic discourse (Peressini & Knuth, 1998; Wertsch & Toma, 1995). That is, students are asked to not only share information but to think about what was said, to incorporate those ideas into their own, and to build upon the conversation in meaningful ways. An additional difference between these final two constructs and their predecessors is the degree to which reflective and instructive communication undergo a shift from predominantly social norms to socio-mathematical norms (Kazemi & Stipek, 1997). That is, teacher and students are no longer focusing on sharing information – a social norm; rather, students and teacher begin to use the discourse to think mathematically, pose conjectures, justify ideas, and generalize – each of which are socio-mathematical norms. Within this process classroom communication becomes instructive when the teacher incorporates students' mathematical ideas and conjectures into the instructional sequence.

Certainly, teachers often use overlapping communication strategies, and as a result it can become difficult to make the subtle distinctions between adjacent constructs as we have defined them. However, we believe one of the contributions of this study is that it does provide insight into the ways in which classroom conversations that have typically been defined into two categories (e.g., univocal and dialogic, Peressini & Knuth, 1998), may be analyzed at a deeper level.

Launching Points for Further Exploration of Mathematical Communication

If we believe that opportunities for students to communicate about mathematical ideas give rise to mathematical understanding, then we certainly need to continue exploring in detail the outcomes – both in terms of students' learning and teachers' development – when these forms of communication are used. Hence, we close this article by articulating several issues that emerged from this research as valuable and necessary points of departure for future work in the area of mathematical communication.

First, as these constructs are explored further, attention must be given to the kind and quality of questions around which these categories are defined and elaborated. Future research in this area should focus closely on the kinds of questions teachers ask within these instructional formats, and the results of those questions on student learning and understanding. For example, it is both easy and tempting simply to suggest that unidirectional teaching can (or should) be avoided by asking more open-ended questions. Yet, merely asking divergent questions does not suggest anything about the quality of the questions in relation to the mathematical content, nor whether the resulting conversations will promote mathematical learning and understanding. Hence, we need to better understand the nature of teachers' questions themselves, along with the kind and quality of the conversation, thinking, and mathematical understanding that they facilitate.

In a related sense, we are in need of further research that explores the boundaries and overlapping regions between the four categories of classroom communication. For example, when does contributive communication actually become reflective for students? What types of questions and teacher behaviors promote this movement from contributive to reflective communication? And, what can we determine about the quality of the reflection in which learners engage? Are teachers consciously aware of conversations that are moving from reflective to instructive levels of discourse? In such instances, what kinds of student comments and concerns truly shape subsequent instruction?

Finally, we should consider the relationship between a teacher's understanding of mathematics and the ability to teach reflectively. As teachers try to establish environments in which mathematical communication and student interactions are encouraged, they are necessarily forced to relinquish control over certain aspects of the classroom. The degree to which teachers – both novice and veteran – are able to work within these open, spontaneous, and dynamic classroom environments, and the impact of this kind of teaching on their emerging beliefs needs to be examined. It is important to consider how preservice teachers' perspectives and knowledge of mathematics, pedagogy, and learning not only change over the initial years of their teaching careers, but also impact the ways in which they create interactive classrooms in which mathematical communication is central.

We offer the case studies of Becky and Brad as an invitation for other scholars in mathematics education to join in contributing to our growing understanding of how mathematics teacher educators might work with teachers, both novice and veteran, in order to create lasting philosophies and practices that foster rich mathematical understanding in learners in our mathematics classrooms.

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