PART 1

THE GOALS OF MATHEMATICAL EDUCATION[1]

George Polya

(As transcribed by Thomas C. O'Brien from a videotaped lecture. The video – 30 plus-years-old – was transcribed thanks to the painstaking technical work of John Ruiz and Steve Berkemeier.)

At a time when university mathematicians are making important decisions about the school mathematics curriculum, perhaps it is appropriate to hear from Professor George Polya. Polya (1887-1985) was a distinguished mathematician and professor at Stanford University who made important contributions to probability theory, number theory, the theory of functions, and the calculus of variations. He was the author of the classic works ***How to solve it*** [1], ***Mathematics and plausible reasoning*** [2], and ***Mathematical discovery*** [3], which encouraged students to become thoughtful and independent problem solvers. He was an honorary member of the Hungarian Academy, the London Mathematical Society, and the Swiss Mathematical Society, and a member of the (American) National Academy of Sciences, the American Academy of Arts and Sciences, and the California Mathematics Council, as well as a corresponding member of the Academie des Sciences in Paris.

The essay that follows is a slightly edited transcript of a videotape lecture that Professor Polya presented to my in-service and pre-service mathematics education students in the late 1960s. I have not corrected Polya's Hungarian-accented language.

T.C. O'Brien

Part 1

What is the aim of teaching mathematics in the primary school?

I wish to talk to you about the teaching of mathematics in the primary school. In fact my talk will consist of two parts. In the first part I will talk about the aims of teaching mathematics in the primary school. And in the second part, how to teach it.

I must confess that I am talking about these things as an outsider. I was always interested in teaching, but most of my time, about half a century, I taught in the university or various universities. And in the last fifteen years, I was mainly concerned with teaching on the high school level. Thus I am talking to you as an outsider, but you may find one or two points in what I am saying that may be useful to you in your profession.

What is the aim of teaching mathematics in the primary school? It is better to consider the most general question: What is the aim of the schools? And the better question is: What do people generally think is the aim of the schools? The first is the point of view of the parents. Your neighbour Mr. Smith has a son Jimmy. He is against Jimmy being a dropout. He says that if Jimmy drops out from school he will never get a right job. So the aim of the school according to Mr. Smith and all the other Mr. Smiths in the general public is to prepare for a job, to prepare the kids to earn a living. But what is the point of view of the community? It is the same. The community, the country, the state, and the city all want people to earn a living and pay taxes and not live on public assistance. So the community also wants the school to prepare the young people to have a job.

If the parents think a little farther, and the community thinks a little farther, the aim is somewhat changed. Reasonable parents, a reasonable Mr. Smith, wants that his son Jimmy should have a job for which he is well fitted. He will earn more and feel happier. By the way, this is also the aim of the community – that you have jobs on one side and people on the other side and you have to assign to people such jobs that totally they are best fitted, that they produce the greatest output. Or even better, that totally the sum of the happiness should be a maximum. What can the school do for that? The point is that when the kid comes to the school you don't know yet what job will come later,

and you don't know for what job he is well fitted, he is best fitted. So what should we do? We should prepare the youngsters so that they can choose between all possible jobs. They must have a view of the whole world around them to recognize for which jobs they will be well fitted. You can express it many ways. I like the following expression: the schools should develop all the interior resources of the child.

We have therefore two kinds of aims in the schools. We have good and narrow aims. The schools should turn out employable adults – adults who can fill a job. But a higher aim is to develop all the resources of the growing child in order that he can fill in the job for which he is best fitted. So the higher aim, I express it so, is to develop all the inner resources of the child.

Now what about mathematics teaching? Mathematics in primary schools has a good and narrow aim and that is pretty clear. An adult who is completely illiterate is not employable in a modern society. Everybody should be able to read and write and do some arithmetic, and perhaps a little more. Therefore the good and narrow aim of the primary school is to teach the arithmetical skills – addition, subtraction, multiplication, division, and perhaps a little more, as well as to teach fractions, percentages, rates, and perhaps even a little more. Everybody should have an idea of how to measure lengths, areas, volumes. This is a good and narrow aim of the primary school – to transmit this knowledge – and we shouldn't forget it.

However, we have a higher aim. We wish to develop all the resources of the growing child. And the part that mathematics plays is mostly about thinking. Mathematics is a good school of thinking. But what is thinking? The thinking that you can learn in mathematics is, for instance, to handle abstractions. Mathematics is about numbers. Numbers are an abstraction. When we solve a practical problem, then from this practical problem we must first make an abstract problem. Mathematics applies directly to abstractions. Some mathematics should enable a child at least to handle abstractions, to handle abstract structures. Structure is a fashionable word now. It is not a bad word. I am quite for it.

But I think there is one point which is even more important. Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems. For the higher aims about which I am now talking are some general tactics of problems. To have the right attitude for problems and to be able to attack all kinds of problems, not only very simple problems which can be solved with the skills of the primary school, but more complicated problems of engineering, physics and so on which will be further developed in the high school. But the foundations should be started in the primary school. And so I think an essential point in the primary school is to introduce the children to the tactics of problem solving. Not to solve this or that kind of problem, not to make just long divisions or some such thing, but to develop a general attitude for the solution of problems.

Part two of this lecture is published on page 42 of this issue.

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Footnote

1 Part I of this transcribed lecture appeared in the September 2001 issue of ***ComMuniCator***, the journal of the California Mathematics Council, Volume 26.1.

References

1 G. Polya: ***How to solve it***; Penguin, 1990. ***How to solve it*** was originally published in 1945 has been translated into 17 other languages.

2 G Polya: ***Mathematics and plausible reasoning***; Vols I & II; OUP, 1954

3 G Polya: ***Mathematical discovery***; Vols I & II; Wiley, 1962

Paper Folding Fractions

A suggested proof for Swan's problem posed in MT180

Equation of the longest diagonal is

$y = a - ax/b$ 1.

Equation of blue line is

$y = ax/mb$ 2.

Where these two intersect we have

$a - ax/b = ax/mb$ 3.

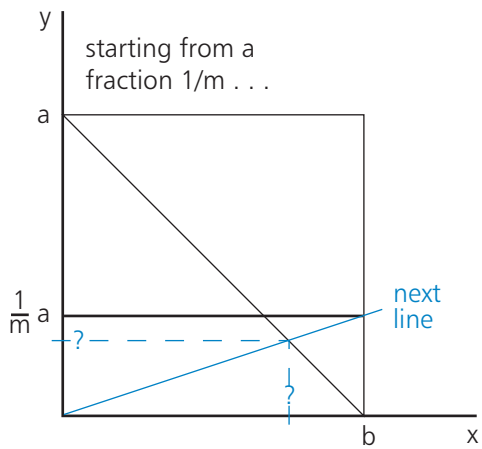
From eq. 3 we re-arrange to get

$x = b/(1 + 1/m)$ 4.

Substitute for x in eq. 1 or 2 and we get

$y = a/(m+1)$ 5.

So the next horizontal after m is m+1.
If we start at 1/3, the next is 1/4, etc.
The series starts at m=1.



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