

# FEEDBACK IN DIFERENT MATHEMATICS TASKS

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*The present article aims to show a research carried out in the context of Project AREA<sup>i</sup>, on the influence that written feedback has in pupil's learning, when given to diversified tasks in the type and in the method of work. The design research was the case study, the participants in the study were four pupils from the 8th year (aged thirteen). The feedback given to tasks of problematic, exploratory or investigative nature seems to be more efficient than in tasks which appeal to the strict knowledge of mathematical concepts. The feedback given to tasks solved in small groups seems to enhance learning in a more significant way when compared to tasks solved individually.*

## INTRODUCTION

At this time a new program of Mathematics for the Elementary School (DGIDC, 2007) is going to start in Portugal. This curricular document comes to reinforce the cross capacities to all the mathematics learning – the problem solving, the mathematical reasoning and the mathematical communication – focusing on the diversification of learning experiences that are offered to pupils over their work in Mathematics, as through tasks of different nature, whether in the form of work between students.

With this research, we intend to study how the written feedbacks, given to pupil's written productions, contribute to their learning process, specifically, taking into account the experiences of learning offered. To do so, we state the following issues:

- What kind of tasks allows feedback as a way to regulate learning?
- Which work methods allow really significant learning, given the teacher's feedback?

## THEORETICAL ARGUMENTATION

Learning results essentially from the activities developed by the pupils and the reflection they do on them (Bishop & Goffree, 1986). To answer to the new standards for mathematics learning (NCTM, 2000) high-level tasks are necessary (Stein, Remillard & Smith, 2007). To achieve learning, teachers must develop processes in order to support students' engagement, in other words, they must attach special importance to the formative assessment. A new approach to learning and its objectives have led to give particular importance to the assessment that aims and contributes for learning. The concept of formative assessment integrated definitely, by legal imposition, the lexicon of the teachers (DGEBS, 1991; Morgan, 2008; DGIDC, 2007). It seems obvious that it is necessary to diversify the type and the

form of assessment instruments, but such is not sufficient for the change of attitudes that a process of assessment for learning imposes (APM, 1995; NCTM, 1995; DGIDC, 2007). Good communication between teachers and pupils is a necessary condition so that the assessment is effectively conducive to learning (NCTM, 1995; 2000; Wiliam, 2007). Therefore, the feedback that teachers give to pupils' productions is a central concept in the formative assessment (Black & Wiliam, 1998; Sadler, 1989), since it allows to students to self-correct their mistakes, improving learning, and to teacher to know better about the pupils difficulties, adapting their practices (Shepard, 2000).

Feedback must happen in the development of the process and in specific aspects of the task (Black & Wiliam, 1998; Schunk & Swartz, 1993; Tunstall & Gipps, 1996), assuming a descriptive and open to dialogue nature (Jorro, 2000; Gipps, 1999). If the feedback focuses in the process rather than the final result and it is complemented with information on the pupils' progress regarding the learning objectives, not only the performance of the pupils but also their beliefs on their competences will improve (Black, Harrison, Lee, Marshall & Wiliam, 2003; Pinto & Santos, 2006). A feedback that differs from the task and focuses on the student tends to have more negative effects, since it approaches aspects that can affect the self-esteem of students (Cameron & Pierce, 1994). The feedback must gives clear guidelines on how the students can improve their production, stimulating reanalysis of the given answers and recognizing what is already well done (Santos, 2003, 2008).

## **METHODOLOGY**

In this research a qualitative/interpretative methodology was followed, using the case study as design research (Bogdan & Biklen, 1994; Merriam, 1991). Four pupils from the 8th year were selected (aged thirteen) whom had already been pupils of the teacher/researcher in the previous academic year. The pupils should show different levels of performance in mathematics, though registering some type of evolution at the level of their commitment and performance in mathematics. Albertina was a student with a low achievement in mathematics and not always committed in overcome her difficulties. Tiago was a student with a low achievement in mathematics, but committed in overcome his difficulties. Manuel was a student with a medium achievement, but just committed with the school tasks. Ricardo was a student with a high achievement in mathematics and very committed.

To gathered data we used the observation, the interview and the analysis of documents, where a log book carried out by the teacher/researcher was included. During the classes in which the pupils developed activities in the context of this research, the teacher was a participant observer. The teacher prepared the tasks to be proposed to the pupils and wrote the feedback to their productions. During the research we carried out three interviews to each one of the pupils, not only with the purpose of gathering their opinions and perspectives about mathematics, mathematics as a school subject and assessment, but also to gather information that might not be

observable during the lessons, namely how the pupils understand the feedback and the reasons that support their options at the time of the second phase. The observations and the interviews were audio recorded and totally transcript. The first and second students' written productions concerning each one of the tasks proposed, as well as the feedback given to the first phases were gathered as documentation.

Data were analyzed considering each type of task (bibliographical inquiry, tests, investigation task and problems) and the way they were worked by the students (individual or small group). In each one, we analyzed the evolution of the pupils' performance from the first phase to the second of each task and of the first task to the second one of each type.

In the context of this research, the eight tasks that were prepared gave origin to twenty productions in two phases (with the exception of the second bibliographical inquiry that was carried out in an unique phase): eight tests, eight problems, two bibliographical inquiries and two investigation tasks, which meets the recommended by NCTM (1995) on the diversity of the type of tasks which must be given feedback. The type of assignments and the method of work are set out in chart 1:

Tasks	Work method
Bibliographical inquiry on Pitágoras	Group
Test in two phases	Individual
Investigation task: study of the influence of k and b in the graphic representation of functions of the type $y=kx+b$	Group
Problem solving	Individual
Bibliographical inquiry on Diofanto	Group
Test in two phases	Individual
Investigation task: divisions by 11	Group
Problem solving	Individual

Chart 1: Types of tasks and methods of work

In all the tasks the pupils had a first moment in classroom to begin its development. After this time, the productions were analyzed and commented. The productions were then delivered to the pupils so that they continued them (Santos, 2008). The second phases of all the tasks were always developed in the next lesson after the end of the first phase, with the exception of the second test, whose second phase was developed a week later. In general, feedback with incidence in the task in analysis was supplied (Black & Wiliam, 1998), in order that each pupil improved in each one of the tasks. Therefore, we tried to use a descriptive feedback (Tunstall & Gipps, 1996) as

dialogue (Jorro, 2000) between the teacher and students, trying to lead them to reflect about each aspect of each task.

## **DATA ANALYSIS**

### **Bibliographical inquiries.**

The bibliographical inquiries constituted, along with the tests, the most closed tasks given because the scripts supplied to the pupils were very directive.

The feedback provided to the bibliographical inquiries consisted on short comments. It was mentioned the unfinished of some aspects, the existence of not too explicit information, the need of developing and of clarifying some information. The decisions taken by the pupils were negotiated between them, sharing the meanings that each one was giving to the written comments by the teacher, so that they could define how to proceed:

Tiago: What do we have?

Ricardo: So, are we going to take the Pythagoras Theorem?

Tiago: Do we take it away?

Manuel: Why?

Ricardo: I am asking.

Tiago: What the teacher wanted to tell is that in order to have here the Pythagoras Theorem we should be putting it. Here like work, as we did with the tree. Do you get it?

When the feedback was not sufficient in order that the pupils improve any aspects, they developed strategies to reach this objective, resorting, for example, to the search of different sources of information.

When they developed the second bibliographical inquiry, the pupils used, by themselves, both phases of the first bibliographical inquiry that they had developed. They shown to have learnt to structure a work of this type, what each part had and which type of information they should gather. Specifically, they improved the conclusion of the work, developing a small summary of what they had done. From the first to the second bibliographical inquiry only some orthographic mistakes persisted. The quality of the second bibliographical inquiry was such that there was not a second phase. Thus, we think that there was a quite positive evolution between the first and the second moment of producing a bibliographical inquiry.

### **Tests.**

The given feedback to the pupils productions were as short comments, clues were given on mathematical contents which the students could use, it was asked in order that they improved justifications and aspects of the tests were set out – this explanation could assume the interrogative form with straight or indirect questions, or the affirmative form. This feedback worked like a progress of the oral questions done

in the lessons in which the students improved routine tasks. It was the “push” that they need to overthrow some trouble felt in this resolution and that was preventing from showing other knowledge. All the students improved their productions from the first to the second phase. Nevertheless, there was not an evolution on the performance of the students from the first to the second test. For example, the fact that the teacher has given a feedback in the first test with the intention of setting out information on the test, so that the student managed to correct his/her answer, it was not enough for the student to understand the whole test in the second test. The two following examples illustrate feedback given to the two tests from the same student. In both, the teacher makes reference to the context of the statement, explaining it:

- 1                   What can you do if you want to call the police, for example?
- 2                   How can you write the number of matches, using the number of the picture?

It was as if everything started again.

### **Investigation tasks.**

The supplied feedback to the investigation tasks assumed nearly always the form of short comments, though in some cases several instructions were given in the same text. In the comments, was indicated the incompleteness of some aspects and questions placed near incorrect aspects. Showing what was already well done, using examples of the pupils to question on aspects of their answer, a few times on a more straight form, other times less, and asked for a deeply explanation of some aspects.

In the first task, the students faced the multiple questions as feedback like being to answer. They did it in the straight questions, but not in the questions that had as objective to lead them to re-analyze their answer to detect mistakes. So, some aspects of the written report produced maintain wrong.

The most obvious improvement in the performance of the group from the first investigation task to the second one was the fact that the students created more examples to help them to reach a generalization. In the first report, when it was asked the students to replace  $k$  by five different values so that they would analyze its influence in the respective graphic representation of the function  $y=kx+b$ , the students did it. However, they did not try any more values besides the five presented, even with difficulties in understanding this influence. In the second task, in spite that in the text were given six fractions with denominator 11, the pupils wrote an orderly sequence of 20 fractions with denominator 11. Grouping them in accordance with criteria chosen by themselves, as illustrates the image 1.

Image 1: Sample of the report

### Problems.

The supplied feedback to the pupils productions were long comments, especially in the first problem. Was highlighted what was already well done, set out conditions that the pupils were not respecting, given clues on mathematical contents were needed to study. These were supplied in the affirmative or interrogative form, using pupils and teacher examples in the direction of setting information out. Students were encouraged to improve their written productions.

When the pupils solved the second problem, a very big evolution was noticed regarding the first problem. This evolution was well-known in the justifications of the reasoning, in the sequential presentation of the attempts done to solve the problem, as well as on the inclusion of some attempts, which came to reveal false ones, accompanied by the explanation for their abandon. The following image is illustrative in this respect:

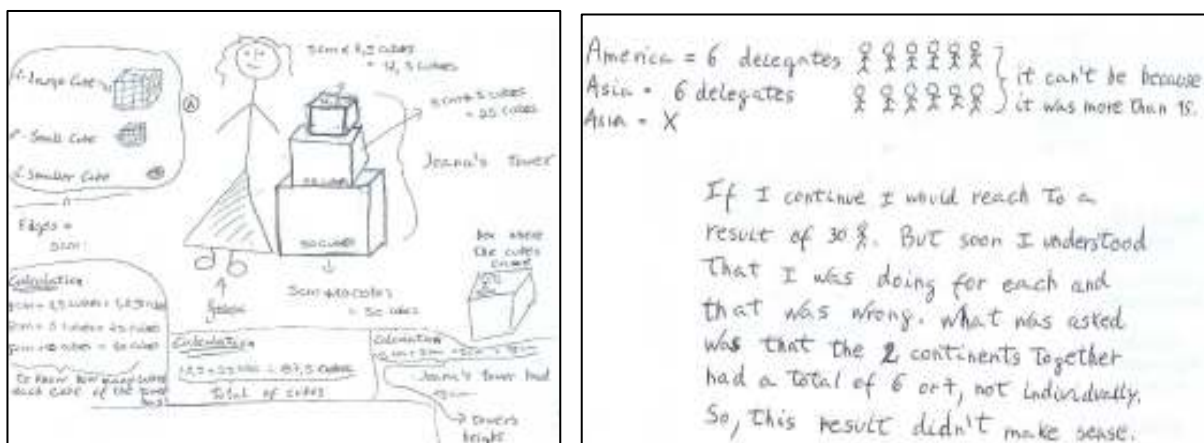


Image 2: Sample of the 2 problems solved by the same student

### CONCLUSIONS

This study point out that the feedback that always tried to be descriptive (Gipps, 1999; Tunstall & Gipps, 1996) and with incidence in the task in analysis (Black & Wiliam, 1998) seems to be more efficient in evolutive terms when it is given to task that appeals to mathematics' capacities as problem solving, mathematical reasoning

and communication. When the task appeals more to the mathematical knowledge, the feedback can be useful for the improvement of the same task. But in a subsequently task of the same type, if the pupils do not dominate the new contents, it is as if everything started again. Students revealed to learn to structure and to understand what was expected that they did in a bibliographical inquiry, started to organize their attempts and to increase the studied cases, to present strategies, even those who came to be revealed unsuccessful and improved the justifications of their strategies and reasoning. The tests, although all of them have improved from the first to the second phase, there are no known improvements of performance from the first one to the second test. The feedback supplied to the tests helped to realize that the aspects of the routine application of not dominated concepts by the pupils were surpassed momentarily, but not in the constant form (Veslin & Veslin, 1992).

The productions carried out in group and that subsequently to the first phase received feedback, were always improved in the second phase and in the second moment they always revealed important improvements in the group performance. The tasks carried out individually which received feedback, not always gave significant improvements in the next phase. Thus, the productions of individual resolution seem not to favor significant learning in comparison with the resolution tasks in group, given the feedback from the teacher. The possibility of pupils confront the interpretations that each one gives to a supplied feedback and to discuss between themselves its meanings seems to constitute a positive factor for the efficiency of the feedback.

Although the feedback given to different tasks provided learning at different levels, we believe that this diversity of tasks contributes to a more solid learning. Therefore, any type of task is to exclude from the learning process. It is necessary to continue to invest in the practice of giving feedback to all kind of students' productions in order to realize what kind of feedback is most appropriate for each type of task.

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<sup>i</sup> Project financed by FCT, nº PTDC/CED/64970/2006.